9400

Bibl. Jag.

IV

82/53 I A 2 1

Praca laboratoryjna

w Glasgowie 1896/7

(a może Cambridge

1905/6 ?)

Die Gesetze des osmotischen Druckes lassen sich daher in kolloiden Lösungen ebensowohl nach zwei Methoden erforschen, welche beide statistischer Natur sind: mittels Beobachtung der Größe der Konzentrationsschwankungen oder der Sedimentationsverteilung im Schwerefeld. Erstere bietet jedoch den erheblichen Vorteil, daß sie auf jede kolloide Lösung mit sichtbaren Teilchen anwendbar ist, während letztere nur für gleichkörnige Hydrosole gute Resultate geben kann. auf Grund

Anfangs schien es, als ob die Erfahrung dem widersprechen würde, indem Svedberg u. Inouye sowie Westgren mittels der Schwankungsmethode an einer ganzen Menge verschiedener kolloider Lösungen sehr bedeutende Abweichungen vom Boyleschen Gesetz konstatierten, während Westgren das exponentielle Gesetz bei der Sedimentation ganz gut bestätigt fand. So betrug z. B. in einem Goldsol (Radius $a = 91\,\mu\mu$) bereits bei einer Volum-Konzentration von nur ~~1,20~~ der Wert $\frac{\beta}{\beta_0} = 0{,}677$; in einer Gummiguttlösung ($a = 200\,\mu\mu$) war $\frac{\beta}{\beta_0} = 0{,}405$ für eine Volum-Konzentration $3{,}8 \cdot 10^{-4}$.

/, /u /a /5.10^{-6} /a

Andererseits hat aber Constatin in Perrins Laboratorium bei Gummiguttlösungen ($a = 30\,\mu\mu$) nach beiden Methoden bis zu weit größeren Konzentrationen vollständige Übereinstimmung mit der idealen Kompressibilität konstatiert und hat erst bei Volum-Konzentrationen von über $^1/_{100}$ eine merkliche Verminderung des β erhalten, welche die Existenz einer Abstoßungssphäre um die Gummiguttteilchen erweist, und zwar müßte letztere eine solche Wirkung ausüben, als ob der Teilchenradius 1,7 mal größer wäre als in Wirklichkeit.

/ß Tß ⊥a

ist

Durch die von Westgren unlängst hier in Göttingen ausgeführte Arbeit ist dieser Widerspruch endgültig aufgeklärt worden, in dem der-

26/3 fresh surface oil lamp

5h 31. 56

~~355~~

5h 37. 55

zero × 351
353) dark (48)

401

♂ × 353 ← ~~42~~ 15 (47)

400

♂ 353 ← 44 5 (48)

401

5h 48.40 light on ♂ 1 min.

zero 355

49.40 ×

50.40 light shining (45.5)

400.5

5h 52.25 ♂ light on 1 min.

535.5

53.25 × (45.0)

54.25 light off

400.5

5h 55.45 ♂ light on 5 min.

6h — 45 × 535.5
401.0 } 45.5

6h 2.55 ♂ light on } 15 min.

6h 17.55 354

18.55 light stopped } 46.3

400.3

6h 32.45 (×) 354 light on

33. 399.7 stop 45.7

6h 36.5 × 353.5 light on

399 45.5

---

fresh surface

6h 47.40 × 352.5 light on

398 45.5

6h 53.25 ×

352.0	42
394	43.5
437.5	42.5
480	42
522	42
564	42
606	41
647	40.5
687.5	40.5
728	40
768	39
807	39
846	38
884	39
923	38
961	

light still on

7^h 11 30 ~~zero~~ 354'5
×

12 30 light stopped

399'5
450

7^h21.15 354.– glow lamp

22.15 459. stopped

27/3 zero 380 12^h8.– glow lamp shining the whole night

379 9.–

12^h10.15 → × 380. oil lamp

11.15 440 ?

when insulated going on by itself. (× and charged up)

12^h 15.15 × 385 dark

393 8

401 8

408'5 7'5 } insulation ?

18.15 408'5

18.30 (410'5) light on

19.30 460 50

510 50

561 51

611 50

661 50

711 50

708 47

806 48

852 46

899 47

945 46 — light put out

948

947'5 no insulation

947'5 loss

12^h36.10

389'5 × dark

396.0 6'5

402'5 6'5

408'0 5'5

460.
409'5
(50'5)

12^h39.25 409'5 light on

40.25 " off

41.25 464.0 } 4.0

42.25 468.0 } 3.5

471'5

~~light on~~

12^h45.40 389.– × dark

394.– 5.–

398 4.–

45.55 light on

46.55 " off

47.55 451.0 } 4.

455.0 } 3'5

458'5

447.–
399
(48.)

12^h54.– 386.– × dark

390.– 4.–

394.– 4.–

55.15 light on

446.–

448.5

451'5

443
395
(48)

27/3 Hg connected to 5

1h 4.30 x 382 light on

382.5

fresh surface

1h 10.30 x 382 dark

386

390.5

12 45 light on

13 45 off

440.8

444

447

437
391.5
45.5

Hg cooled by water

1h 44.40 x 372 dark

374

380

47.40 383

1h 47.55 → light on

48.56 off

49.56 430

435

435

1h 54.30 371.5

375

378

56.45 light on

57.45 off

426

427.5

428.5

glow light on the whole time from 2h
zero slowly creeping up

3

4h 40 50 zero x 366 light on

41 " 417.3 off

42 " 420.

414.5
366
48.5

light on

4h 46.5 zero x 372

47.5
48.5 422.5 light off

49.5 424.3

420.3
372.0
48.3

fresh surface

4h 52.50 372.7

53.50

54.50 419.2

55.50 421.–

422.8

417.5
372.7
44.8

fresh surface ~~hea~~ (warmed up by hand)

5h 58 20 363 } light on

59.20
60 20 411.5

415.–

408.5
363
45.5

6h 19.20 ~~293~~

366 dark | 40 Volts

366+

2 min. — light

441

442

440
366
74 : 2 = 37

6h 26.30 | 365 } 1 min. light

27.30

409

410.5

407.5
365
42.5

| 6.4 Volts on

6h 33 364 } 1 min. light 28

392

392

6h 39.45 365 | 4.2 Volts on

389.5 24.5

389.5

6h 44.40 365 | 2.1 Volts on

377 12.0

6h 47.20 365

377 12.0

~~5h 38 365~~

4.2 Volts on

6h 53.30 366

390.5 24.5

~~6h 57 46 365~~ 6.4 Volts on

7h 0.30 366 } 1 min light

396.

30.

7h 4.40 365 40 Volts

411 414.–

365

46 417.

120 Volts

7h 13 .20 × 292

289.5

288.5

Needle charge 120 Volts

7h 36.20 590 6.4 Volts on

639

642.5

645

636
590
46
43

7h 42 15 × 592 4.2 Volts on

594.5

598

20 637.5

634.5
598.5
36

28/3

120 V.

7^h 53 45 x 595 dark

601

606

612

618

light on — 620

55

— 675

685

55

10 — 10

695

45

28/3 1^h 6 40 zero 360·5 dark

x 360.0

360·2

360·3

360·2

360·2

360·1 without P.D.

1^h 9 40
10.20

glow lamp on

360·7

361·7

362·3

363·1

363·3 off

363·4

1^h 17.20 363·5 light on

4^h 10 365·3

zero 350.

4.24 50 raised so that part of glass surface fresh

357

416

fresh surface

366 45

411

4.36.50 362·5 52

414·5

fresh surface

~~4.41.~~ - 365

410.5 45·5

shone on

4.56.7 364

518.2

54.2

~~fresh surface~~

360

409

459

508

557

28/3

353
344
391
395
1/2 398

454

355
410) 55
409

356
408'5) 52'5

2 min light

359
411'5 52'5

29/3 glow lamp on solid crystals of Na Amalgam

5	354	42'5
	396'5	39'5
	436	39
light shining	475	36
the whole time	511	

347 38
385

350	47
397	39'5
436'5	38'5
475	36
511	

[illegible]

488

586

617

[illegible] zero 352

[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	50
[illegible]	50
[illegible]	4[illegible]
[illegible]	4[illegible]
[illegible]	[illegible]6
[illegible]	46
[illegible]	[illegible]
[illegible]	46
81[illegible]	45
[illegible]	[illegible]
907	45
9[illegible]2	
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	50
[illegible]	43
[illegible]	[illegible]
[illegible]	[illegible]
5[illegible]3	43

[illegible] 421

11h 0 a.m. 11h 56

[illegible]	47
416	
465	49
513	54
[illegible]	52
[illegible]	50
[illegible]	[illegible]
[illegible]	52
[illegible]	41
170	51
[illegible]	[illegible]
159	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	44
[illegible]	[illegible]
[illegible]	56
422	53
[illegible]	[illegible]
585	50
635	50
685	[illegible]
280	[illegible]
240	43
[illegible]	51
[illegible]	50
934	

zero 334

460	45
502	42
[illegible]	40
944	
828	
944	116
[illegible]	47

[illegible]

4.5'2

[illegible] during the whole time [illegible]

[illegible]

11 : 1.5 = 52 32' [illegible]

5[illegible]

[illegible]

56

[illegible] 27 : [illegible]

4[illegible] 40

533

6[illegible] [illegible] 41

6[illegible] 40

7[illegible] 3[illegible]

[illegible] 17 [illegible] 3[illegible]

[illegible] 77 : [illegible] = [illegible]

[illegible]

Durch die von Westgren unlängst hier in Göttingen ausgeführte Arbeit ist dieser Widerspruch endgültig aufgeklärt worden, in dem derselbe nachwies, daß die nach Svedbergscher Methode mit Hilfe des Spaltultramikroskops ausgeführten Teilchenzählungen bei hell leuchtenden Teilchen eine subjektive Fehlerquelle enthalten[1]), welche bei größerer Konzentration sehr stark ins Gewicht fällt und jene Abweichungen vorgetäuscht hatte, während die Zählungen, welche Westgren an einem zwischen Deckgläsern eingeschlossenen Präparat ausführte, noch für eine Goldsuspension ($\phi = 110\ \mu\mu$) von einer Konz. $= 2{,}3 \cdot 10^{-4}$ vollkommen normale Kompressibilität ergaben. Nach Analogie mit Costantins Resultaten zu schließen, wären Anomalien tatsächlich erst bei etwa 50 mal größeren Konzentrationen zu erwarten, welche sich in reinen Goldhydrosolen überhaupt kaum herstellen lassen dürften.

1a

Das bisher Besprochene bezog sich auf die Abweichungen vom Boyleschen Gesetz, welche natürlicherweise vor allem die damit zusammenhängenden Erscheinungen der Schwankungsgröße

---

1) Undeutliche Definition des beleuchteten Volums infolge seitlicher Zerstreuung des Lichtes. Ob dies auch für die von Lorenz u. Eitel an Tabakrauch gefundenen Abweichungen gilt, ist wohl erst durch weitere Untersuchungen zu entscheiden. Literaturnachweise, siehe S. [redacted] sowie die Zusammenstellung bei Th. Svedberg, Jahrb. d. Rad. u. Elektr. 10, 467, 1913.

Insbesondere hat Paine nachgewiesen, daß die Koagulationszeit, in Übereinstimmung mit unseren Formeln, umgekehrt proportional der Anfangskonzentration des Kolloids ist und daß sie umgekehrt proportional der 5. oder 6. Potenz der Elektrolytkonzentration variierte, was wir einer entsprechenden Änderung des Wirksamkeitskoeffizienten ε zuzuschreiben haben.

Letzteres kann aber natürlich kein allgemeines Gesetz sein, da bei stärkeren Zusätzen, in dem oben besprochenen Bereich der „raschen“ Koagulation, die Koagulationsgeschwindigkeit von der Elektrolytkonzentration unabhängig wird. Sehr instruktiv sind in dieser Beziehung einige Zahlen, welche mir R. Zsigmondy gütigst mitgeteilt hat, wonach zur Erreichung eines bestimmten, an dem Farbumschlag Rot- Rotviolett kenntlichen Koagulationsgrades einer Goldlösung bei verschiedenen Elektrolytkonzentrationen c (Millimol $NaCl$ pro Liter) die nachstehenden Zeiten T (Sekunden) erforderlich waren:

c	5	10	20	50	100	150	200	300	500
T	> 150	12	7,2	7	7	6	6,5	7,5	7[1]

Ein dem Anfang dieser Messungsreihe angepaßtes Potenzgesetz müßte natürlich bei höheren Konzentrationen vollständig versagen.

Auch Freundlichs und Ishizakas Messungen bestätigten, wie gesagt, das Ähnlichkeitsprinzip, und zwar in bezug auf die Abhängigkeit vom Elektrolytzusatz, sonst sind sie aber zu einer quantitativen Kontrolle unserer Formeln nicht geeignet, da sie sich (wie auch Paines Messungen) nicht auf die Teilchenzahlen selbst, sondern auf andere Größen bezogen, wie Zähigkeit oder in anderen Untersuchungen gewisse Adsorptionserscheinungen, welche komplizierte und einstweilen unbekannte Funktionen der Zahl und Größe der Teilchen (bzw. deren Aggregate) sind.

Macht man betreffs der Abhängigkeit der

1) Allerdings könnte auch R von der Elektrolytkonzentration abhängen. Formell ist das aber mit einer Änderung des ε gleichwertig.

Somit könnte man annehmen, daß in Paines Versuchen alle Teilchen abgeschieden wurden, welche aus mehr als k-Primärteilchen bestanden (wo k eine große Zahl ist), während der Rest, bestehend aus

$$L = \nu_1 + 2\nu_2 + 3\nu_3 \ldots (k-1)\nu_{k-1}$$

Primärteilchen, in Lösung blieb. Werden hierin unsere Formeln (70) eingesetzt, so ergibt sich für die nicht koagulierte Menge, bei Benutzung der Abkürzung $\varepsilon\beta t = \alpha$, der Ausdruck:

$$L = 1 + \frac{(k+\alpha)^{k-1}}{(1+\alpha)^k},$$

welcher für große k gleichwertig ist mit:

$$L = 1 + \left[1 + \frac{1}{x}\right] e^{-\frac{1}{x}} \tag{74}$$

wo x zur Abkürzung für die zur Zeit proportionale Größe

$$x = \frac{\alpha}{\nu} = \frac{\varepsilon\beta t}{\nu}$$

eingeführt ist.

Zeichnen wir uns diese Koagulationskurve auf, so überzeugen wir uns, daß sie tatsächlich ganz überraschend ähnlich mit den von Paine erhaltenen empirischen Kurven verläuft. Insbesondere muß eine ~~große~~ Zeit, die „Inkubationszeit“, verstreichen, bevor sich die Teilchen soweit vergrößern, daß überhaupt ein merklicher Niederschlag erhalten wird, dann tritt bei dem Werte gewisse

$x = \frac{1}{3}$, $\cancel{t} = 0{,}801$ ein Wendepunkt auf und von L b

da an verläuft die Kurve konvex nach abwärts, um sich asymptotisch der Zeitachse anzuschmiegen. Der Unterschied besteht nur darin, daß der Übergang von der Inkubationszeit in die Koagulationskurve hier etwas allmählicher erfolgt als in Paines Kurven und die scharfe unnatür- m

~~Hypotheses and mental experiments~~

truthfulness — anti-vagueness of statement

passion for accuracy, ~~passion~~ for facts — suspended judgment

~~fanaticism~~ of exactitude — clearness of vision

~~enthusiasm~~ for truth

sense of inter-relatedness

biography Maxwell
Faraday etc.

Cognition p. 50
law of nature = descriptive formula

there may be psychological & social generalisation which really tell us why that...

but chemical and physical — only how

Huxley : science = game
rules of game

drudgery Joule mech. equivalent
Andrews . . . water.
Rayleigh Argon

|| polarised in agreeing

Kelvin p. 75 Natural history - Natural philosophy stage

und einstweilen unbekannte Funktionen der Zahl und Größe der Teilchen (bzw. deren Aggregate) sind.

Macht man betreffs der Abhängigkeit der Viskosität von der Teilchenzahl und -größe gewisse, ziemlich plausible Annahmen, so kann man die charakteristische, durch einen Wendepunkt gekennzeichnete Gestalt der Koagulationskurven Freundlichs ohne weiteres erklären, doch kommen da zu viel hypothetische Elemente ins Spiel, als daß man von einer zahlenmäßigen Kontrolle reden könnte und deshalb will ich auf diese Rechnungen hier nicht weiter eingehen.

Auch scheint es mir nicht rationell, aus derartigen Messungen die Differentialgleichung der Koagulationskinetik ableiten zu wollen, wie es Freundlich versucht, solange man nicht weiß, wie die Teilchenzahlen mit dem beobachteten Effekt zusammenhängen. Für erstere, nicht aber für den gemessenen Gesamteffekt, ist eine einfache Gesetzmäßigkeit zu erwarten. Dagegen sind derartige Messungen wohl geeignet, auf Grund des Ähnlichkeitsprinzips hochinteressante Aufschlüsse über die Abhängigkeit der Übergang von der Inkubationszeit in die Koagulationskurve hier etwas allmählicher erfolgt als in Paines Kurven und die scharfe unnatürliche Ecke derselben vermeidet. pn

Wenn man diese wenigen bisher zu Gebote stehenden Kontrollversuche überblickt, gewinnt man wohl den Eindruck, daß die in Rede stehende Verallgemeinerung unserer, den Koagulationsmechanismus auf Brownsche Bewegungen zurückführenden Theorie dem Wesen der Sache entspricht, und man kann wohl hoffen, daß dieselbe sich als Wegweiser bei weitergehenden Untersuchungen auf diesem bisher der Mathematik ganz unzugänglichen Gebiete nützlich erweisen dürfte. 1/

(Eingegangen 1. September 1916.)

1) Die Abweichungen von 7 Sek. bei 50—500 Millimol liegen innerhalb der Beobachtungsfehler.

[I Ziele u. Wege d. math. Unter.] Im Sinne von Klein, 12

gewinnt durch [illegible]

§1 Inhaltliche & formale Bildg. d. d. Math.

Meraner Vorschläge Stärkg d. räuml. Anschauung [illegible] & funktionalen Denkens

1884 eher Inhalt als Form

§2 Funktionales Denken & Anschauung

§3 Unterbau im math. U. : Fertigkeit im Rechnen & Zeichnen

„selbst auf Kosten d. Gründlichkeit"; scheinbare Ungründlichkeit = Voraussetzung d. didaktischen [illegible]

§4 Reine u. angewandte Math.

Das Konkrete vor d. Abstrakten

zuerst etwas zeigen, dann darüber reden

§5 Pädagogisch-didakt. Darbietung: verboten „Setzen Sie einmal [illegible] langweilig"

§6 Grundsätze d. Lehrplangestaltung: Anpassung an die psychologischen Gegebenheiten d. Schülerseele

[illegible], didaktische Einheiten

II Lehrproben, Lehrgänge, Lehrpläne

gegliedert nach Lebensalter

Einheitslehrplan für Gymn. & Realsch. aller Länder, [illegible]

Vorallem erste Einführg ausführlich besprochen, [illegible]

§7 Zur Charakteristik d. Unterstufe

dem 10-13 jährigen Kinde gibt es noch keine math. Wissenschaft [illegible]

[illegible] logische Formulierung beibringen soll, bringt nur leere Worte bei, keine Gedanken

Zeichnen lernen! Raumlehre, Anschauen

§8 Zur Vorschule d. Zahlenlehre [illegible] an die Volksschule anknüpfen

[illegible]: Individuelle Klassen, dann [illegible] Zahlenkreis
einzelne Zahlen als Individuen

arithm. Lehrtext wie er nicht sein soll

[illegible] Kritik: logische Fehler, Definitionssucht

in Wirklichkeit sollen die Kinder lernen die Wörter (Terminologie) zu gebrauchen, aber ohne dass Definitionen gegeben werden.

Lehrtextprobe, [illegible] wirkliches Zählen!

§9 D. mechanische Rechnen in d. untersten Klassen

fester Mechanismus [illegible], deutlich sprechen etc.
wie Griffe beim Exerzieren

§10 Dezimalzahlen, gemeine Brüche, Dezimalbrüche [illegible] p. 79
[illegible] räumliche Anschauung

§11 Weiteres: gegen Proportionen
aber wohl ~~[illegible]~~ Gedanke d. Proportionalität Textprobe aus [illegible] p. 86

§12 Zur Vorschule d. Raumlehre

geom. Lehrt. wie er nicht sein soll   ausführliche Kritik

§13 Skizzen zu einer Vorsch. ...
[illegible]

§14 [illegible] Symmetrie, Kongruenz

Übungen im Gelände, Konstruktionen [illegible] Betätigung

§15 Buchstaben rechnen Zerlegung mittels Linienstücke

Abgekürztes Rechnen, Irrationalzahlen Pythagoreischer Lehrsatz

Mittelstufe 14, 15 J.

§16 Euclid nur einige Beispiele zur Illustration d. Methode etc.

Gegen sog. wissensch. Lehre von d. Rechnungsoperationen

Dagegen viel Gleichungen u zwar „Textgleichungen"

§17

§18 Erziehung des Anschauens für Logik der Strenge aber nicht Maßstab

19 Nicht mit Sätzen über Lagen der Geraden u Ebenen an zu fangen sondern Vorkursus d. Stereometrie

§20 Darstellende Geometrie! Elemente Zeichnen von Krystallformen Perspective

§21 Vol. von [illegible] Körpern, Cavalierisches Prinzip (Heron v. Alexandr.)

Oberstufe 16–18 J.

§22 Funktionsbegriff

Grenzbegriff, complexe Zahlen etc. = [illegible]

§23 Lehrsätze über Potenzen
§24 Logarithmen
} Verbindg mit graph. Bd. Funktionsbegriff

Lehraufgabe d. Trigonometrie

§30 Für Oberklassen a. [illegible] [illegible] :

|| Prinzip d. Beweisgrg d. angewandten.
Gegensatz d. reinen ~~Math~~

Weg mit Formalismus!

Excurse

Digressionen

[illegible]

[illegible]

gerufen wird. Aber die Erscheinungen des reversibeln Koagulationsgleichgewichts, welche z. B. Sven Odén an Schwefelhydrosolen studiert hat, gehen über den Geltungsbereich dieser Theorie vorläufig noch hinaus. Sie bildet also selbstverständlich nicht eine allseitige Aufklärung des ganzen Problems, sondern nur einen ersten Schritt auf diesem seitens theoretischer Begründung noch vollständig unbe~~kann~~ten Gebiete.

|bau

Vor allem lassen sich schon daraus gewisse Schlüsse ziehen, daß die Koagulation in unserer Theorie auf die Brownsche Molekularbewegung und auf die Existenz einer Wirkungssphäre R zurückgeführt wird, denn als Variable, von welchen der Koagulationsverlauf abhängt, kommen somit nur folgende drei in Betracht: der Radius R, die Teilchenzahl pro Volumeinheit n_0 und die Diffusionskonstante D, deren Dimensionen gegeben werden durch das Schema:

$$D \sim \frac{l^2}{t}; \; n_0 \sim \frac{1}{l^3}; \; R \sim l.$$

1) Eine ausführlichere Darstellung wird in der Zeitschr. f. phys. Chem. veröffentlicht werden.

1. Das hervorgehobene Teilche[n] und für sich eine ähnliche Brow[nsche Be]wegung aus wie die übrigen, es ko[mmt] für die Koagulation die relative Bewe[gung in] Betracht. Diesbezüglich läßt sich nu[n] nachweisen, daß die Relativverschiebung [zweier] sich unabhängig voneinander bewegen[der Teil]chen ebenso erfolgt, wie die gewöhnlich[e Brown]sche Bewegungsformel (1) angibt, nur [mit dem] Unterschied, daß der Diffusionskoe[ffizient] gleich der Summe der Koeffizienten [beider] Teilchen zu setzen ist. Allgemein g[ilt für] die Relativbewegung:

$$D_{ab} = D_a + D_b .$$

2. Unsere Formeln (61) (62) en[tsprechen] dem Falle, wo die Zahl ν_0 der Tei[lchen in] größerer Entfernung unverändert ble[ibt. In] Wirklichkeit kleben sie aber nicht nur [an dem] hervorgehobenen, sondern auch unter[einander] an. Von der Anzahl $4\pi D R \nu_0$ sind a[lso jene] wegzustreichen, welche schon vor der [Zeit] koaguliert sind, somit ist die Zahl ν_0 du[rch die] Zahl ν_1 der zur Zeit t noch existierenden [Einzel]teilchen zu ersetzen. Ebenso kommen als [Koagula]tionskerne, wenn es sich um die Verei[nigung] von einfachen zu Doppel-Teilchen handel[t, nicht] sämtliche ν_0 sondern nur die noch frei[en Teil]chen ν_1 in Betracht.

$$\delta = \frac{2\pi}{\lambda}\left[\frac{h}{v_0\cos\beta_0} + \frac{h(\tan\beta_e - \tan\beta_0)\sin\alpha}{c} - \frac{h}{v_e\cos\beta_e}\right]$$

$$= \frac{2\pi h}{\lambda}\left[\frac{\cancel{M}\sin\alpha}{c}(\tan\beta_e - \tan\beta_0) + \cdot\right.$$

$$+ \frac{2\pi h}{\lambda}\left[\frac{h}{v_0} - \frac{h}{v_e} + \frac{h(\beta_e-\beta_0)\alpha}{c}\right]$$

$$\left[\frac{1}{v_0}\left[1+\frac{\beta_0^2}{2}\right] - \frac{1}{v_e}\left[1+\frac{\beta_e^2}{2}\right] + \frac{(\beta_e-\beta_0)\alpha}{c}\right]$$

$$v_e = o\left[1 - \frac{\alpha^2}{2}\,\frac{e^2(o^2-e^2)}{c^2o^2}\right] \qquad \beta_e = \frac{e^2}{oc}\alpha$$

$$\beta_0 = \frac{o}{c}\alpha$$

$$\frac{1}{o}\left[1+\frac{o^2\alpha^2}{2c^2}\right] - \frac{1}{o}\left[1+\frac{\alpha^2}{2}\,\frac{e^2(o^2-e^2)}{c^2o^2}\right]\left[1+\frac{e^4}{o^2c^2}\frac{\alpha^2}{2}\right] + \left[\frac{e^2}{oc}-\frac{o}{c}\right]\frac{\alpha^2}{c}$$

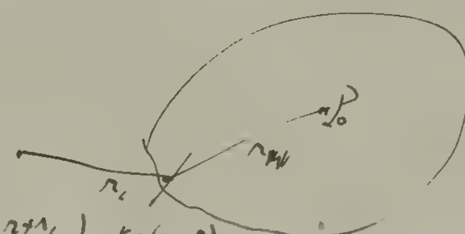
$$= \alpha^2\left\{\cancel{\frac{o}{2c^2}} - \frac{e^2(o^2-e^2)}{2c^2o^3} - \frac{\cancel{e^4}}{2\cancel{o^3}c^2} + \frac{e^2}{oc^2} - \frac{o}{2c^2}\right\}$$

$$= -\left[\frac{e^2}{oc^2} + \frac{o}{c^2}\right]\frac{\alpha^2}{2}$$

$$4\pi U_0 + \int \left[\frac{1}{r}\frac{\partial U}{\partial n} - \cos(n\,r)\,\frac{\partial}{\partial n}\left(\frac{U}{r}\right)\right] dS = \int \frac{1}{r}\left(\nabla^2 U - \frac{\partial^2 U}{\partial t^2}\right) dv$$

$$U = \frac{A}{r_1}\cos 2\pi\left(\frac{t}{\tau} - \frac{r_1}{\lambda}\right)$$

$$\frac{\partial^2 U}{\partial t^2} \neq \frac{\partial^2 U}{\partial t^2} = \nabla^2 U$$



$$U_0 = \frac{1}{4\pi}\int \frac{1}{r}\frac{\partial U}{\partial n}\cos n\,r_1$$

$$\frac{\partial U}{\partial n} = \frac{2\pi A}{\lambda r_1}\sin 2\pi\left(\frac{t}{\tau} - \frac{r+r_1}{\lambda}\right)\cdot\cos(n\,r_1)$$

$$\frac{\partial}{\partial n}\left(\frac{U}{r}\right) = \frac{2\pi A}{\lambda r r_1}\sin 2\pi\left(\frac{t}{\tau} \dots\right)$$

$$4\pi U_0 = \frac{2\pi A}{\lambda}\int \frac{1}{r r_1}\sin 2\pi\left(\frac{t}{\tau} - \frac{r+r_1}{\lambda}\right)\cdot\left[\cos n\,r + \cos(n\,r_1)\right] dS$$

$\sigma_x'' = \sigma_e' \sin\varphi - \sigma_0' \cos\varphi$

$= a \sin\varphi\cos\varphi \sin\vartheta - \sin\ldots$

$1+\cos\delta = 2\cos^2\frac{\delta}{2}$

$- a \cos\varphi \cos(\psi-\varphi) \sin u + \sin\varphi \sin(\psi-\varphi) \cos(u+\delta)$

$= a \sin u\left[\cos\varphi \cos(\psi-\varphi) + \sin\varphi \sin(\psi-\varphi) \cos\delta\right] + \cos u\left[\sin\varphi \sin(\psi-\varphi) \sin\delta\right]$

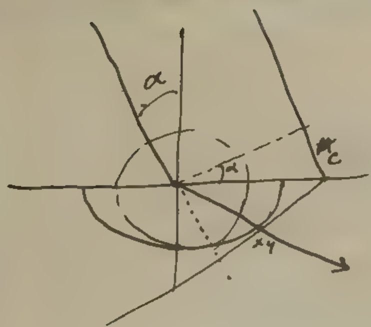
$A^2 = \cos^2\varphi \cos^2(\psi-\varphi) + \sin^2\varphi \sin^2(\psi-\varphi) + \underbrace{2\sin\varphi\cos\varphi \sin(\psi-\varphi) \cos(\psi-\varphi)}_{\frac{1}{2}\sin2\varphi \sin2(\psi-\varphi)\cdot\cos\delta} \cdot \cos\delta$

$\underbrace{\pm 2 \sin\varphi \cos \ldots}$

$\underbrace{\left[\sin\varphi \sin(\psi-\varphi) + \cos\varphi \cos(\psi-\varphi)\right]}_{\cos^2\psi}^2 - 2\sin\varphi\cos\varphi \ldots \quad (1-\cos\delta)$

$\cos^2\psi - \sin2\varphi \sin2(\psi-\varphi) \sin^2\frac{\delta}{2}$

$= \cos^2\psi + \sin2\varphi \sin2(\varphi-\psi) \sin^2\frac{\delta}{2}$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x\xi}{a^2} + \frac{y\eta}{b^2} = 1$$

$$\xi = \frac{c}{\sin\alpha}$$

$$\eta = 0$$

$$x = \frac{a^2\sin\alpha}{c}$$

$$y = b\sqrt{1-\frac{x^2}{a^2}} = b\sqrt{1-\frac{a^2\sin^2\alpha}{c^2}}$$

$$\sin\beta = \frac{x}{\sqrt{x^2+y^2}} = \frac{\frac{a^2\sin\alpha}{c}}{\sqrt{\frac{a^4\sin^2\alpha}{c^2}+b^2-a^2b^2\frac{\sin^2\alpha}{c^2}}} = \frac{a^2\sin\alpha}{\sqrt{b^2c^2+a^2(a^2-b^2)\sin^2\alpha}}$$

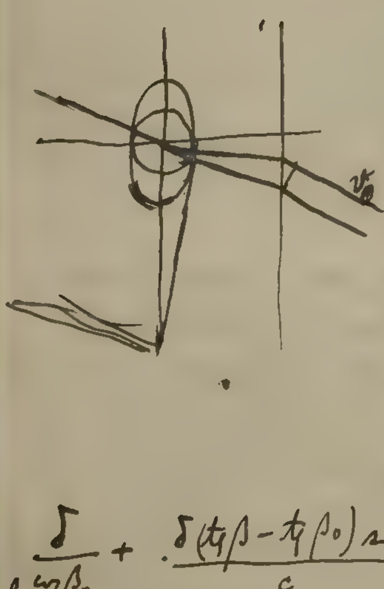
$$\sin^2\beta\left[b^2c^2+a^2(a^2-b^2)\sin^2\alpha\right] = a^4\sin^2\alpha$$

$$\sin^2\beta\cdot b^2c^2 = a^2\left[a^2\cos^2\beta + b^2\sin^2\beta\right.$$

$$\frac{\sin\alpha}{\sin\beta} = \frac{bc}{a\sqrt{a^2\cos^2\beta+b^2\sin^2\beta}}$$

$$= \sqrt{b^2-\frac{b^2a^2}{c^2}\sin^2\alpha+\frac{a^4}{c^2}\sin^2\alpha}$$

$$= \sqrt{b^2-\frac{a^2(b^2-a^2)}{c^2}\sin^2\alpha}$$



$$\frac{\delta}{\cos\beta}$$

$$r_\beta = \sqrt{x^2+y^2} = \frac{x}{\sin\beta} = \frac{a^2\sin\alpha}{c\sin\beta}$$

$$= \frac{\delta\cdot c\sin\beta}{a^2\sin\alpha\cos\beta} = \frac{\delta\, a\sqrt{a^2\cos^2\beta+b^2\sin^2\beta}}{a^2 b\cos\beta}$$

$$= \frac{\delta}{b}\sqrt{a^2+b^2\tan^2\beta} \;\ldots$$

$$\frac{\delta}{b\cos\beta_0} + \frac{\delta(\tan\beta-\tan\beta_0)\sin\alpha}{c}$$

$\underline{X} = \underline{Y} = 0$ zatem $\underline{N} = 0$ $\qquad$ $\cancel{\frac{\partial X}{\partial x}} + \cancel{\frac{\partial Y}{\partial y}} + \frac{\partial Z}{\partial z} = 0$

$$\frac{\partial M}{\partial t} = \frac{\partial Z}{\partial x}$$

$$\frac{\partial^2 Z}{\partial t^2} = a^2 \left(\frac{\partial^2 Z}{\partial x^2} + \frac{\partial^2 Z}{\partial y^2} \right)$$

$$\frac{\partial L}{\partial t} = -\frac{\partial Z}{\partial y}$$

Z tylko zmienne w kierunkach x z

zatem kierunek największej zmienności wzdłuż w oś $\underline{X}$: $\frac{\partial Z}{\partial y} = 0$

$$\frac{\partial^2 Z}{\partial t^2} = a^2 \frac{\partial^2 Z}{\partial x^2}$$

$N = L = 0$ $\qquad$ $\cancel{\frac{\partial L}{\partial x}} + \frac{\partial M}{\partial y} + \cancel{\frac{\partial N}{\partial z}} = 0$

$$\frac{\partial^2 M}{\partial t^2} = a^2 \frac{\partial^2 M}{\partial x^2} \longleftarrow \frac{\partial X}{\partial t} = \frac{\partial M}{\partial z} - \cancel{\frac{\partial N}{\partial y}}$$

$Z = A \sin \alpha \left(t - \frac{x}{a}\right)$ $\qquad$ $M = B \sin \alpha \left(t - \frac{x}{a}\right)$

$\left.\begin{matrix} X \\ Y \end{matrix}\right\} = 0$ $\qquad$ z tego zaraz wypływa: $L = N = 0$

$$\frac{\partial M}{\partial t} = -\frac{\alpha A}{K_\mu a} \cos \alpha \left(t - \frac{x}{a}\right)$$

$$M = \frac{A}{K_\mu} \sin \alpha \left(t - \frac{x}{a}\right)$$

K elektromagnetycznie mierzone?:

$$\kappa \frac{\partial X}{\partial t} - - = \frac{\partial \beta}{\partial y}$$

$$\dim K = \frac{H}{E} \qquad \frac{1}{\sqrt{\mu K}} = v$$

$$\dim \frac{K_m}{K_e} = \underset{\overset{\shortparallel}{v^{-1}}}{\frac{H_m}{H_e}} \cdot \underset{\overset{\shortparallel}{v^{-1}}}{\frac{E_e}{E_m}} = v^{-2}$$

ogólnie : ~~$\sqrt{\mu_m K_e} = \sqrt{\mu_m K_m} \cdot \sqrt{\frac{K_e}{K_m}}$~~

$$c^2 = \frac{1}{\mu_m K_m} \cancel{= \frac{1}{\mu_m K_e} \cdot \frac{K_e}{K_m}}$$

K_e dla eteru $= 1$

$(K_m = v^{-2}$

$$= \frac{1}{\mu_m K_m} = \frac{1}{\mu_m K_{em}} \cdot \frac{K_{em}}{K_m}$$

$$\left(\frac{c}{v}\right)^2 = \frac{1}{n^2} = \frac{1}{K^2}$$

$$K \frac{\partial \mathfrak{E}}{\partial t} + 4\pi\lambda \mathfrak{E} = \operatorname{curl} \mathfrak{H} \qquad \frac{\partial}{\partial t} \Big| \quad \mu \frac{\partial \mathfrak{H}}{\partial t} = -\operatorname{curl} \mathfrak{E} \quad \ldots$$

$$\mu K \frac{\partial^2 \mathfrak{E}}{\partial t^2} + 4\pi\lambda\mu \frac{\partial \mathfrak{E}}{\partial t} = \nabla^2 \mathfrak{E}$$ tak samo dla $\mathfrak{H}$

$$\frac{\partial^2 Z}{\partial t^2} + \frac{4\pi\lambda}{K} \frac{\partial Z}{\partial t} = \cancel{\ } a^2 \nabla^2 Z$$

$$Z = a e^{-\gamma x} \sin \alpha \left(t - \frac{x}{a}\right)$$

$$Y = X = 0$$

Jeżeli λ duże w porówn z ...

to tak jak przewodnictwo ciepła

$\gamma =$

$a =$

...

$$\sigma = r\sin\varphi + \alpha\rho\delta\sin(\varphi+\varepsilon) + \alpha\rho^3\delta\sin(\varphi+2\varepsilon) + \dots$$

$$= r\sin\varphi + \alpha\rho\delta\{\sin\varphi[\cos\varepsilon + \rho^2\cos 2\varepsilon + \dots]$$

$$+ \cos\varphi[\sin\varepsilon + \rho^2\sin 2\varepsilon + \dots]\}$$

$$e^{i\varepsilon} + \rho^2 e^{2i\varepsilon} + \dots = \frac{e^{i\varepsilon}}{1-\rho^2 e^{i\varepsilon}} = \frac{\cos\varepsilon + i\sin\varepsilon \quad (\dots)}{(1-\rho^2\cos\varepsilon - \rho^2 i\sin\varepsilon)(1-\rho^2\cos\varepsilon + \rho^2 i\sin\varepsilon)}$$

$$= \frac{\cos\varepsilon(1-\rho^2\cos\varepsilon) - \rho^2\sin^2\varepsilon}{(1-\rho^2\cos\varepsilon)^2 + \rho^4\sin^2\varepsilon} + i\,\frac{\sin\varepsilon(1-\rho^2\cos\varepsilon) + \rho^2\sin\varepsilon\cos\varepsilon}{\dots}$$

$$= \frac{\cos\varepsilon - \rho^2}{1-2\rho^2\cos\varepsilon+\rho^4} + i\,\frac{\sin\varepsilon}{1-2\rho^2\cos\varepsilon+\rho^4}$$

$$\sigma = \sin\varphi\left[r + \alpha\rho\delta\,\frac{\cos\varepsilon-\rho^2}{1-2\rho^2\cos\varepsilon+\rho^4}\right] + \cos\varphi\;\alpha\rho\delta\,\frac{\sin\varepsilon}{1-2\rho^2\cos\varepsilon+\rho^4}$$

$$J = r^2 + 2\alpha\rho\delta r\,\frac{\cos\varepsilon-\rho^2}{1-2\rho^2\cos\varepsilon+\rho^4} + (\alpha\rho\delta)^2\,\cancel{\frac{\rho^4 + \rho^2\cos\varepsilon + 1}{[1-2\rho^2\cos\varepsilon+\rho^4]^4}}$$

$: \quad 1 - 2\rho^2\cos\varepsilon + \rho^4 \pm 2\,(1-r^2)\cos\varepsilon \mp 2r^2(1-r^2) + (1-r^2)^2$

$\alpha^2 = 1-r^2$

$\cos\varepsilon = 0$ $\quad 4r^4$

$\cos\varepsilon = -1, +1$ $\quad 4(1-r^2)^2$

$$= 2 - 4r^2 + 4r^4 + 2\cos\varepsilon - 4r^2\cos\varepsilon = 2\{\cos^2\tfrac{\varepsilon}{2} - 2r^2\cos^2\tfrac{\varepsilon}{2} + 2r^4\}$$

$$= \cos^2\tfrac{\varepsilon}{2}(1-2r^2) + 2r^4$$

wenn $\alpha\delta\rho = -$ $\quad \dfrac{2 \qquad\qquad -2\cos\varepsilon}{1-2\rho^2\cos\varepsilon+\rho^4} = 4\sin^2\tfrac{\varepsilon}{2}\,r^2$

~~$\alpha\delta\rho = +$: $(1-r^2)^2 + 1 + \dfrac{2(1-r^2)[\cos\varepsilon-\rho^2]}{1-2\rho^2\cos\varepsilon+\rho^4} = 2 - 2r^2 + r^4$~~

$= \cancel{(\cos\Delta + i \sin\Delta)} = \cancel{\sin^2\varphi +}$

$$\alpha = \frac{1+\beta}{1-\beta}$$

$$\alpha - \alpha\beta = 1+\beta$$

$$\beta = \frac{\alpha-1}{1+\alpha} = -\frac{1-\alpha}{1+\alpha}$$

$$\frac{1-e^{i\Delta}}{1+e^{i\Delta}} = -\,i\,\frac{\cos\varphi\sqrt{\sin^2\varphi - n^2}}{\sin^2\varphi}$$

$$\frac{(1-e^{i\Delta})(1-e^{-i\Delta})}{-\ -} = \frac{1-e^{i\Delta}-e^{-i\Delta}+1}{-\ -}$$

$$= \underbrace{\frac{1-\cos\Delta}{1+\cos\Delta}}_{=\operatorname{tg}\frac{\Delta}{2}\,=} = \left[\frac{\cos\varphi\sqrt{\sin^2\varphi-n^2}}{\sin^2\varphi}\right]^2$$

$$\left.\begin{array}{l}\varphi = \frac{\pi}{2}\\ \sin\varphi = n\end{array}\right\}\ \Delta = 0$$

$$\frac{\partial}{\partial\varphi} = 0 = \frac{2n^2 - \sin^2\varphi(1+n^2)}{\sin^3\varphi\sqrt{\sin^2\varphi - n^2}}$$

$$\sin^2\varphi' = \frac{2n^2}{1+n^2}$$

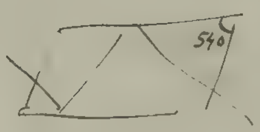
$$\operatorname{tg}\frac{\Delta'}{2} = \frac{1-n^2}{2n}$$

$\text{sklo} \,\|\, n = \frac{1}{1{\cdot}51}$

$\Delta' = \ldots0^\circ 76'$

dla $\varphi = 48^\circ 37'$

$54^\circ 37'$ $\quad \Delta = 45^\circ$



Dla $n<1$

$$\cos\varphi = -i\sqrt{\frac{\sin^2\varphi}{n^2}-1}$$

Jakie znaczenie ampl. uroj.?

$$\sigma = A\sin\alpha\overbrace{\left(t-\frac{x}{v}\right)}^{\varphi} = A\,\frac{e^{i\varphi}+e^{-i\varphi}}{2i} = \mathcal{I}\left(A e^{i\varphi}\right)$$

to samo też można zrobić w razie jeśli A urojone

$$A = A_0 e^{i\delta} \qquad \sigma = A_0\,\mathcal{I}\left[e^{i(\varphi+\delta)}\right]$$

więc różnica fazy δ

$= A_0\sin(\varphi+\delta)$

tak samo

$\sin\varphi$

$\sin(\varphi+\pi) = -\sin\varphi$

$\sin(\varphi+k\pi) = (-1)^k\sin\varphi$

$\sin(\varphi+\frac{\pi}{2})$ $(-1)^{1/2}\sin\varphi$

$(\alpha+i\beta)\sin\varphi = \alpha\sin\varphi + \beta\cos\varphi$

$$A_0 e^{i\delta}\sin\varphi = (A\cos\delta + iA\sin\delta)\sin\varphi = a(\cos\delta\sin\varphi + \sin\delta\cos\varphi) = a\sin(\varphi+\delta)$$

$$B_s e^{i\delta_s} = \frac{\frac{i\cos\varphi}{\sqrt{\sin^2\varphi-n^2}}+1}{\frac{i\cos\varphi}{\sqrt{\sin^2\varphi-n^2}}-1}A_s$$

$$B_p e^{i\delta_p} = \frac{\frac{i\cos\varphi\cdot n}{\sqrt{\cdots}}+\frac{1}{n}}{\frac{i\cos\varphi\, n}{\sqrt{\quad}}-\frac{1}{n}}A_p$$

składając $A_s = A_p$

$(-\alpha+1)(\alpha+1) = 1-\alpha^2$

$(\alpha-1)(-\alpha-1) = 1-\alpha^2$

Z drugiej strony $B_s e^{-i\delta_s} = \frac{+\frac{\cos^2\varphi}{\sin^2\varphi-n^2}+1}{\frac{\cos^2\varphi}{\sin^2\varphi-n^2}+1}A_s = A_s$

$B_s = A_s$

$$e^{i(\delta_s-\delta_p)} = e^{i\Delta} = \frac{\left[\frac{i\cos\varphi}{\sqrt{\quad}}+1\right]\left[\frac{i\cos\varphi\cdot n}{\sqrt{\quad}}-\frac{1}{n}\right]}{\left[\frac{i\cos\varphi}{\sqrt{\quad}}-1\right]\left[\frac{i\cos\varphi\cdot n}{\sqrt{\quad}}+\frac{1}{n}\right]} = \frac{\cancel{\frac{n\cos^2\varphi}{\sin^2\varphi-n^2}}-\frac{1}{n^2}}{}$$

$\frac{n^2(1-n^2)-\sin^2\varphi}{n} = \sin^2\varphi\,\frac{n^2-1}{n}$

$$= \frac{\left[i\cos\varphi+\sqrt{\quad}\right]\left[in\cos\varphi-\frac{1}{n}\sqrt{\quad}\right]}{-} = \frac{-n\cos^2\varphi-\frac{\sin^2\varphi-n^2}{n}+i\left\{n\cos\varphi\sqrt{\quad}-\frac{1}{n}\cos\varphi\sqrt{\quad}\right\}}{-}$$

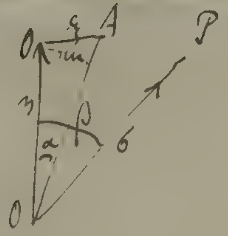
$+$

$$= \frac{\sin^2\varphi+i\cos\varphi\sqrt{\quad}}{\sin^2\varphi-i\cos\varphi\sqrt{\quad}} = \frac{1+\frac{i\cos\varphi}{\sin^2\varphi}\sqrt{\quad}}{1-\cdots} =$$

$$\sin \varepsilon_1 = \frac{\beta_1}{\sqrt{\alpha_1^2+\beta_1^2}} \qquad \sin \varepsilon_2 = \frac{\beta_2}{\sqrt{\alpha_2^2+\beta_2^2}}$$

$$\cos \varepsilon_1 = \frac{\alpha_1}{\sqrt{\alpha_1^2+\beta_1^2}} \qquad \cos \varepsilon_2 = \frac{\alpha}{\sqrt{- -}}$$

$$\operatorname{tg}^2 \frac{\Delta}{2} = \frac{1-\cos\Delta}{1+\cos\Delta} = \frac{1 - \frac{\alpha_1\alpha_2+\beta_1\beta_2}{\sqrt{(\alpha_1^2+\beta_1^2)(\alpha_2^2+\beta_2^2)}}}{1 + \ldots}$$



$$\xi = a \sin\gamma \sin\alpha$$

$$\eta = a \sin\gamma \cos\alpha$$

$$\sigma = a \sin(\varphi - \delta_0) \cancel{\sin\alpha \sin(\beta-\alpha)} + a \sin(\varphi - \delta_2) \cos\alpha \cos(\beta-\alpha)$$

$$\frac{\psi^2}{a^2} = \left[\sin\alpha \sin(\beta-\alpha) + \cos\alpha \cos(\beta-\alpha) \cos\varepsilon\right]^2 + \left[\cos\alpha \cos(\beta-\alpha) \sin\varepsilon\right]^2$$

$$= \left[\sin\alpha \sin(\beta-\alpha) + \cos\alpha \cos(\beta-\alpha)\right]^2 + \left[\cos\alpha \cos(\beta-\alpha) \sin\varepsilon\right]^2 + 2 \underbrace{\sin\alpha \cos\alpha \sin(\beta-\alpha) \cos(\beta-\alpha)}_{\sin 2\alpha \sin 2(\beta-\alpha)} \left[\cos\varepsilon - 1\right]$$

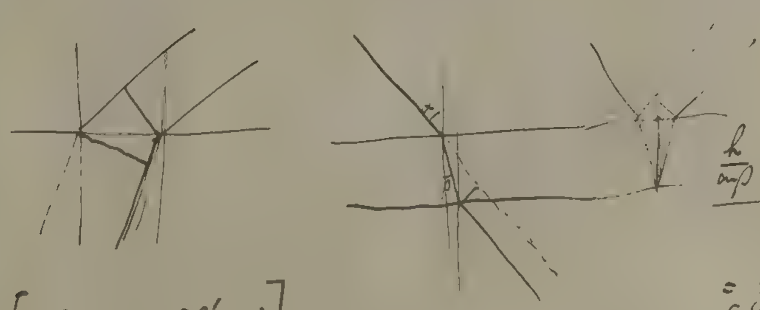
$$/\cos^2\beta \qquad \sin^2\frac{\varepsilon}{2}$$

$$K_1^2 + K_2^2 + 2 K_1 K_2 \cos\delta \qquad K_1 = \cos\alpha \cos\beta$$

$$= (K_1 + K_2)^2 - 4 K_1 K_2 \sin^2\frac{\delta}{2} \qquad K_2 = \sin\alpha \sin\beta$$

$$J = \cos^2(\beta-\alpha) + \sin^2 2\alpha \sin^2\beta \ldots \frac{\delta}{2}$$

$$\alpha = \beta \qquad J_0 = 1 - \sin^2 2\alpha \sin^2\frac{\delta}{2} \qquad \Big| \; \alpha \perp \beta \qquad \sin^2 2\alpha \sin^2\frac{\delta}{2}$$



$$\frac{\frac{h}{\cos\beta}\cos(\alpha-\beta)}{c} - n\,\frac{\frac{h}{\cos\beta}}{c}$$

$$= \frac{h}{c\cos\beta}\left[\cos\beta \cos(\alpha-\beta) - \cancel{n \cos\beta}\right]$$

$$\cos\alpha\cos\beta\sin\beta + \underbrace{\sin\alpha \sin^2\beta - \sin^2\alpha}_{\sin 2\alpha \ldots}$$

$$= \cos\beta(\cos\alpha\sin\beta - \sin\alpha\cos\beta)$$

$$= \cos\beta \sin(\beta-\alpha)$$

$$\frac{h}{c}\left[\sin(\beta-\alpha) - \sin(\beta'-\alpha)\right]$$

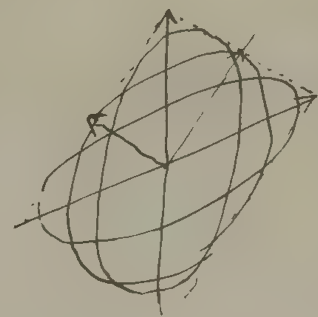
$$\cancel{\sin\beta\cos\alpha - \cos\beta\sin\alpha}$$

$$(\sin\beta - \sin\beta')\cos\alpha - \cancel{\sin\alpha}(\cos\beta - \cos\beta')$$

$$\mp \left(\frac{\varrho}{c} - \frac{\varrho^2}{\sigma c}\right)\sin\alpha$$

Drugi sposób powstawania: geometryczny

W tych wszystkich razach mówimy jest światło eliptyczne lub cyrkularne



$\cancel{y = b \sin \alpha t}$

$x = a \sin \alpha t$

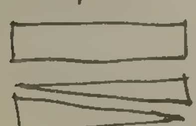
$y = b \sin(\alpha t + \delta) = b \cos\delta \sin \alpha t + b \sin\delta . \cos \alpha t$

$y - \frac{b}{a} x \cos\delta = b \sin\delta \cos \alpha t$

$$\left[\frac{y}{b \sin\delta} - \frac{x}{a \,\mathrm{tg}\, \delta}\right]^2 + \frac{x^2}{a^2} = 1$$

$$\frac{x^2}{a^2 \cancel{tg} \cos^2\delta} - \frac{2xy \,\mathrm{tg}\,\delta}{ab} + \frac{y^2}{b^2} = \sin^2\delta$$

Aby dowolne różnice fazy wytworzyć Babinet Compensator



$\frac{1}{4}$ Und. Glimmer

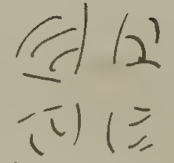
$\cdot/ =$

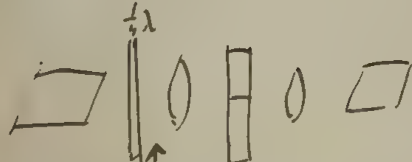
$X_n \cos x + Y_n \cos y + Z_n \cos z$

Für $\psi = \frac{\pi}{2}$

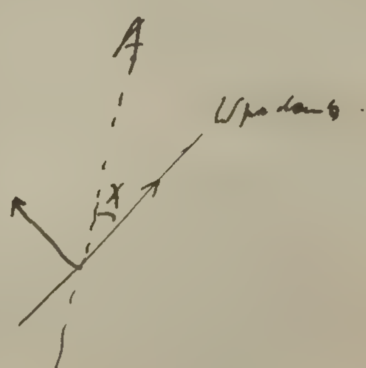
$$A^2 = + \sin^2 2\psi \; \sin^2\frac{\delta}{2}$$

$\psi = 0$: $$A^2 = 1 \mp \sin^2 2\psi \; \sin^2\frac{\delta}{2}$$





$(\psi_0) = \xi = a \sin\frac{2\pi}{\tau} t$
$(6_0) = \eta = a \cos\frac{2\pi t}{\tau}$



$$6 = \cancel{a \sin\frac{2\pi}{\tau} t - \varepsilon_0}$$

$$= a \sin(u-\varepsilon_0)\cos\chi + \cos(u-\varepsilon_0)\sin\chi$$

$$= a\left(\sin u \cos\chi + \cos(u+\delta)\sin\chi\right)$$

$$= \sin u\,(\cos\chi - \sin\chi \sin\delta) + \cos u \cos\delta \sin\chi$$

$$A^2 = (\cos\chi - \sin\chi\sin\delta)^2 + (\sin\chi\cos\delta)^2$$

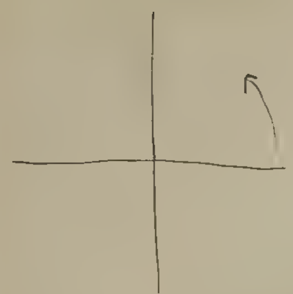
$$= \cos^2\chi + \sin^2\chi - \sin 2\chi \sin\delta$$

$$= 1 - \sin 2\chi \sin\delta$$

$\chi = 45°$: $1 - \sin\delta$
$-45°$: $1 + \sin\delta$

Für $\chi = 0$, $\frac{\pi}{2}$

[illegible]



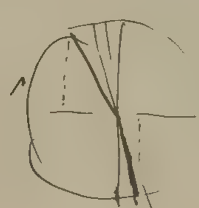
$$\xi_1 = \frac{a}{2} \cos \frac{2\pi}{\tau}\left(t - \frac{x}{\lambda}\right) \qquad \xi_2 = -\frac{a}{2} \cos \frac{2\pi}{\tau} t$$

$$\eta_1 = \frac{a}{2} \sin \frac{2\pi}{\tau} \qquad \eta_2 = \frac{a}{2} \sin \frac{2\pi}{\tau}$$

$$\xi' = \frac{a}{2}\left[\cos \frac{2\pi}{\tau}\left(t - \frac{x}{\lambda}\right) - \cos \frac{2\pi}{\tau}\left(t - \frac{x}{l}\right)\right] = -a \sin \frac{2\pi}{\tau}\left(t - \frac{x}{2}\left(\frac{1}{\lambda} + \frac{1}{l}\right)\right) \sin \frac{\pi}{\tau} x \left(\frac{1}{\lambda} - \frac{1}{l}\right)$$

$$\eta' = \frac{a}{2}\left[\sin \qquad + \sin \qquad\right] = a \sin \qquad \cos \frac{\pi}{\tau} x \left(\frac{1}{\lambda} - \frac{1}{l}\right)$$

$$\frac{\eta}{\xi} = -\operatorname{tg} \frac{\pi}{\tau} x \left(\frac{1}{\lambda} - \frac{1}{l}\right)$$



$\sin(\varphi + \psi) \mid \sin(\varphi - \psi)$

Tak zachowują się kryszt. jednoosiowe (heksagon, tetragonal
1-3 1-2

szpat, kwarzec, ~~...~~, mika, turmalin
apatyt, szmaragd
~~...~~ lód

cyrkon (hyacinth)

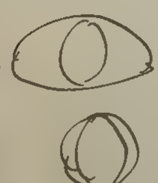
granat, ałun, sól, fluoryt
dyament
(anyal. ... anomalie)

dwuosiowe (trzyosiowe rombowe, jednosk. monokl., dwusk. trikl.)

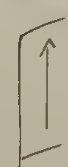
~~...~~	aragonit	gips	$CuSO_4$
	topas	ortoklas	
	siarka	$FeSO_4$	

negat.

pozyt.



Turmalin dichroityczny absorbuje o, przepuszcza tylko e



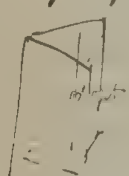
Szybkości światła w kierunkach osi jednoosiowych

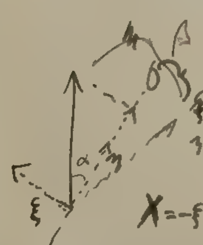
$CaCO_3$ $\frac{v_o}{v} = 0{,}60295$ | $\frac{v_e}{v} = 0{,}67279$

$n_o = 1{,}658$ $n_e = 1{,}487$

SiO_2 $n_o = 1{,}544$ $n_e = 1{,}553$

można mierzyć za pomocą pryzmatu



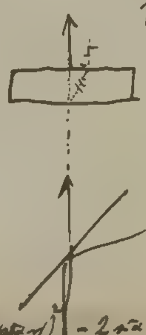


$$\xi = \sin\left(\varphi - \frac{\Delta\, 2\pi}{v_o \tau}\right) \sin\alpha$$

$$\eta = \sin\left(\varphi - \frac{\Delta\, 2\pi}{v_e \tau}\right) \cos\alpha$$

$$X = -\xi \sin\beta + \eta \cos\beta$$

$$0 \quad = \sin(\varphi - \delta) \cos\alpha \cos\beta - \sin\varphi \sin\alpha \sin\beta$$



tylko o przejdzie
e zniknie

Płaszczyzna polaryzacji dla o — pł. wyz. dana wraz z pł. główną

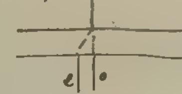
promień o drga $\perp$ do osi optycznej

promień e drga w płaszczyźnie głównej przekroju

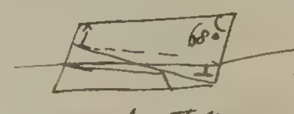
$$A^2 = [\cos\alpha\cos\beta \cdot \cos\delta - \sin\alpha\sin\beta]^2 + (\sin\delta \cos\alpha\cos\beta)^2 = (\cos\alpha\cos\beta)^2 + (\sin\alpha\sin\beta)^2 - 2\sin\alpha\cos\alpha\sin\beta\cos\beta\cos\delta = \cos^2(\alpha+\beta) + \frac{\sin 2\alpha \sin 2\beta}{2}(1-\cos\delta)$$

Podw. załam. wszystkie kryształy z wyjątkiem regular.

polaryzuje się w płaszcz. głów. do płaszcz.



3 razy tak długi jak



w płaszczyźnie głów.

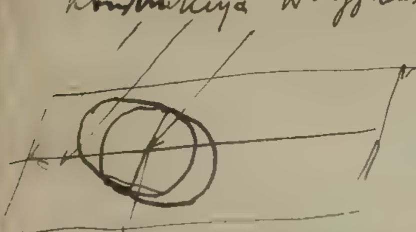
wyj. obu tylko e

W świetle polaryzowanem przez odbicie:

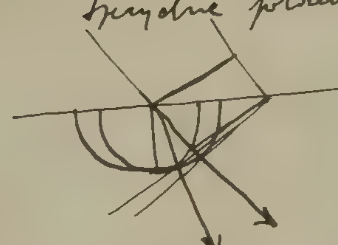
ciemność nikoli gdy płaszcz. wpad. św. = płaszcz. głów.

Erasmus Bartholinus (Duńczyk) 1670

konstrukcya Huyghensa:



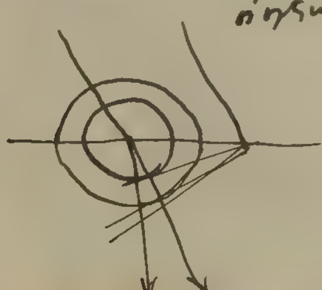
Szczególne położenia



prostują się w płaszcz.

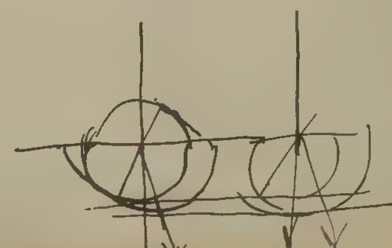
ale

oś optyczna



zwinna oś

Też przy prostopadłem wpadaniu



$$A^2 = (\cos\alpha\cos\beta) + (\sin\alpha\sin\beta)^2 - 2\sin\alpha\cos\alpha\sin\beta\cos\beta\cos\delta =$$

$\|$ równa się $[(\cos\delta-1)^2+\sin^2\delta]$ $\underbrace{2(1-\cos\delta)}_{4\sin^2\frac{\delta}{2}}$

$$= \Big[\underbrace{\cos\alpha\cos\beta - \sin\alpha\sin\beta}_{\cos(\alpha+\beta)^2}\Big]^2 + \sin 2\alpha\sin 2\beta\,\sin^2\frac{\delta}{2}$$

Np. $\beta = -\alpha$

$$A^2 = 1 - \sin^2 2\alpha\,\sin^2\frac{\delta}{2}$$

jeśli $\alpha = 45^\circ$: $A^2 = 1 - \sin^2\frac{\delta}{2} = \cos^2\frac{\delta}{2}$

$$= 1 - \sin^2\frac{\pi\Delta}{\lambda}(n_o - n_e)$$

barwy jak cienkiej blaszki, zależne od Δ

ciemności dla: $\frac{\pi\Delta(n_o-n_e)}{\lambda} = \frac{\pi}{2}$

$$\lambda = \underline{2\Delta(n_o-n_e)}$$

~~[illegible]~~ $\alpha+\beta = \frac{\pi}{2}$

$$A^2 = +\sin^2 2\alpha\,\sin^2\frac{\delta}{2}$$

$\alpha = 45^\circ$: $A^2 = \sin^2\frac{\delta}{2}$

Kręcąc analizat. zmieniają się barwy

$\alpha = 0$: $A^2 = \cos^2\beta$ biało

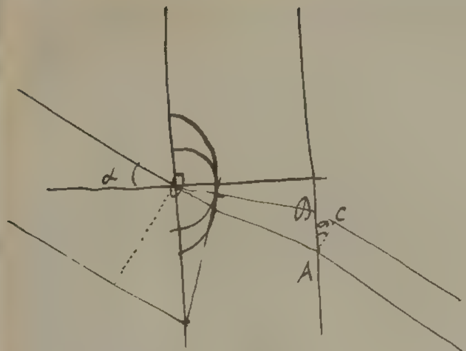
$\alpha = \frac{\pi}{2}$: $A^2 = \sin^2\beta + \sin 2\beta\sin^2\frac{\delta}{2} = \sin\beta[\sin\beta + \cos\beta(1-\cos\delta)]$

$\alpha = \frac{\pi}{4}$: $A^2 = \frac{1}{2}(\cos\beta-\sin\beta)^2 + \sin 2\beta\sin^2\frac{\delta}{2} = \frac{\cos^2\beta+\sin^2\beta-2\cos\beta\sin\beta}{2} + \sin\beta\cos\beta(1-\cos\delta)$

$$= \frac{(\cos\beta+\sin\beta)^2}{2} - \sin\beta\cos\beta\cos\delta$$

To samo powstaje również, jeśli oś nie leży w linii [illegible] nikoli, tylko wtedy ciemności dla δ innej, a zamiast 00 właściwie położenie ~~[illegible]~~ przekroju głównego.

W spektroskopie: ~~[illegible]~~ ciemne prążki w równych odstępach

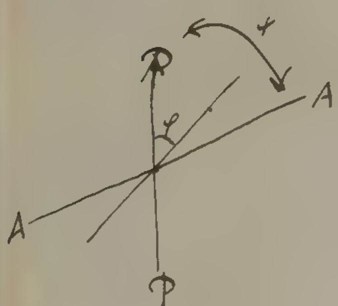


$$\delta = \frac{2\pi}{\tau}\left[\frac{OB}{v_o} + \frac{BC}{c} - \frac{OA}{v_e}\right]$$

$$= f_c(\Delta, \alpha)$$

$$= \cancel{f(\alpha, 0)} + \cancel{\alpha \frac{\partial f}{\partial \alpha}} + \alpha^2 \frac{\partial^2 \varphi}{\partial \alpha^2}$$

$$\left(= \frac{d}{2 v_o}\left(v_e^2 - v_o^2\right)\sin^2\alpha\right)$$



$$\xi = a \cos\varphi \sin\vartheta$$

$$\eta = a \sin\varphi \sin(\vartheta - \delta)$$

$$X = \xi \cos(\psi - \varphi) - \eta \sin(\psi - \varphi)$$

$$= a \sin\vartheta\left[\cos\varphi \cos(\psi-\varphi) - \sin\varphi \sin(\psi-\varphi)\cos\delta\right]$$

$$+ a \cos\vartheta\left[\sin\varphi \sin(\psi-\varphi) \sin\delta\right]$$

$$A^2 = \cos^2\varphi \cos^2(\psi-\varphi) - 2\sin\varphi\cos\varphi \sin(\psi-\varphi)\cos(\psi-\varphi)\cos\delta + \sin^2\varphi \sin^2(\psi-\varphi)$$

$$= \left[\cos\varphi\cos(\psi-\varphi) - \sin\varphi\sin(\psi-\varphi)\right]^2 + 2\sin\varphi\cos\varphi \sin(\psi-\varphi)\cos(\psi-\varphi)(1-\cos\delta)$$

$$= \cos^2\psi + \sin 2\varphi \sin 2(\psi-\varphi)\sin^2\frac{\delta}{2}$$

[illegible] $\psi = 0$: $A^2 = 1 - \sin^2 2\varphi \sin^2\frac{\delta}{2}$

$\psi = \frac{\pi}{2}$: $A^2 = \sin^2 2\varphi \sin^2\frac{\delta}{2}$ $\Big\{ = 0$ dla $\varphi = 0$, $\varphi = \frac{\pi}{2}$ etc.

lub dla $\delta = 2k\pi$

lösungen und den dabei auftretenden theoretischen Problemen brieflich Mitteilung gemacht hatte, habe ich eine mathematische Theorie der Koagulationskinetik ausgearbeitet, welche eine spezielle Anwendung der im Vorhergehenden entwickelten Theorie der Brownschen Bewegung bildet, und diese möchte ich Ihnen heute in einem ganz kurzen Abriß vorlegen[1]).

Von vornherein seien jedoch zwei einschränkende Bemerkungen vorausgeschickt:

1. Meine Theorie beansprucht nicht als vollständige Aufklärung der inneren Ursachen der Koagulation, d. h. der hierbei in Wirkung tretenden elektrischen oder kapillaren Kräfte, der Natur der elektrischen Doppelschichte auf der Oberfläche der Kolloidteilchen usw., zu gelten. Es ist eine sozusagen formale Theorie, aufgebaut auf einer mir von Prof. Zsigmondy vorgeschlagenen Annahme betreffs des Mechanismus der Koagulation, wonach sich jene Kräfte durch eine Wirkungssphäre vom Radius R ersetzen lassen, derart, daß die Brownsche Bewegung der Teilchen ungehindert vor sich geht, solange die Entfernung ihrer Mittelpunkte größer ist als R, daß jedoch zwei Teilchen sofort aneinander haften bleiben müssen, sobald ihre Mittelpunktsentfernung auf R herabsinkt.

2. Eben infolge dieser Annahme bezieht sich diese Theorie eigentlich direkt nur auf einen Grenzfall der Koagulations-Kinetik, d. i. die rasche irreversible Koagulation, wie sie bei größeren Elektrolytkonzentrationen zustandekommt. Ich glaube, daß man sie mittels gewisser Modifikationen teilweise auch auf die langsame Koagulation ausdehnen kann, welche durch geringen, die elektrolytische Doppelschicht nicht vollständig entladenden Elektrolytzusatz hervorgerufen wird. Aber die Erscheinungen des reversibeln Koagulationsgleichgewichts, welche z. B. Syen Odén an Schwefelhydrosolen studiert hat, gehen über den Geltungsbereich dieser

$$\int dt = 4\pi DRc\left[1 + \frac{\quad}{\sqrt{\pi Dt}}\right] dt$$

und für die von Anfang an abgeschiedene M…

$$M = 4\pi DRc\left[t + \frac{2R\sqrt{t}}{\sqrt{\pi D}}\right].$$

Behufs Vereinfachung der Rechn… wollen wir schon an dieser Stelle eine An… rung einführen, indem wir das zweite … der rechten Seite, welches die $\sqrt{t}$ enthäl… unwesentlich weglassen. Das heißt, daß wi… Koagulationsverlauf in einem solchen St… studieren, wo die Zeit t groß ist gegenübe… Werte $\frac{R^2}{D}$. Das Anfangsstadium, welches … diese Bedingung ausgeschlossen wird, … sich beispielsweise in Zsigmondys Vers… auf nur 10^{-4} bis 10^{-3} Sekunden. Im ü… könnte man die Rechnung auch ohne jen… nachlässigung weiterführen, ~~erhält~~ aber … ziertere, praktisch gleichwertige Formeln…

Ersetzt man die Konzentration c du… pro Volumeinheit entfallende Teilchenz… so ~~würde~~ die Anzahl der pro Zeiteinheit a… hervorgehobenen Adsorptionskern ankle… Teilchen: $4\pi RDv_0$, und die Zeit

$$T = \frac{1}{4\pi RDv_0} = \frac{1}{\beta},$$

welche wir weiterhin „Koagulationszeit… wollen, würde dem Zeitpunkt entspre… durchschnittlich gerade ein Teilchen … hervorgehobenen haften bleibt.

3. Vervollständigte Berechn… Koagulation.

Nun ist aber unsere Rechnung i… Hinsicht zu verbessern:

1. Das hervorgehobene Teilche… und für sich eine ähnliche Brow… wegung aus wie die übrigen, es ko… für die Koagulation die relative Bewe… Betracht. …

E.POLLY, IV KAROLINENGASSE 23

$X_1 =$ [illegible] (1)

$X_2 =$ [illegible]

$X_3 =$ [illegible]

$Y_1 =$ [illegible]

$Y_2 =$ [illegible]

$Y_3 =$ [illegible]

$$\frac{d}{dt} = \frac{1}{[\ldots]^{2}} [\ldots]$$

$$- r_1^{3} - 3 m_2 x_2 r_2 (x_1 - x_2) - 3 m_3 x_3 r_3 (\ldots) \ldots$$

$$- (\ldots)(m_2\, 3 r_2 (x_1 - x_3) \ldots m_3\, 3 r_3 \ldots (x_1 - x_2))$$

$$\frac{d x_1}{dt^2} +$$

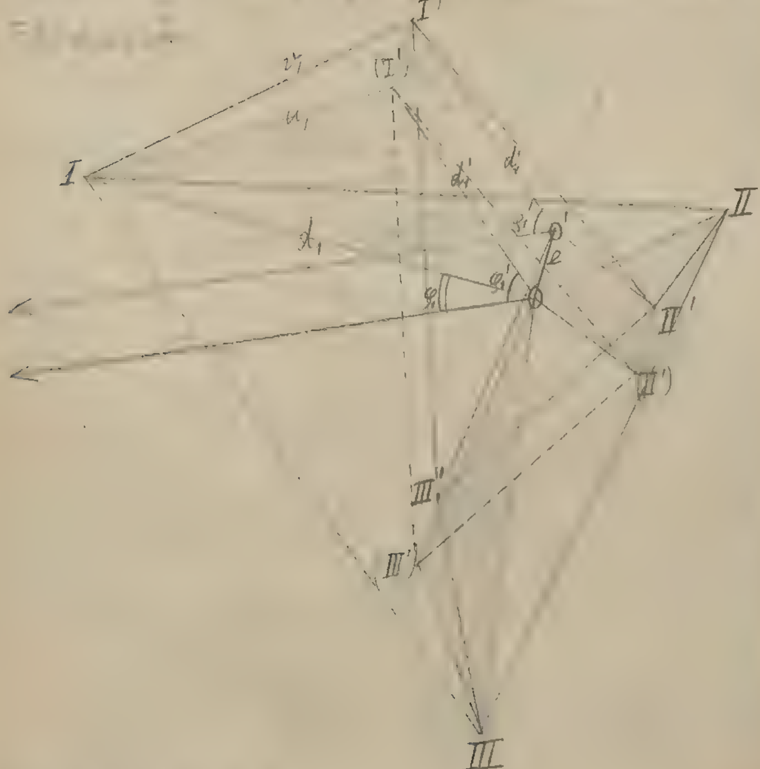
$$= \frac{1}{[\quad]^2} [\ldots]$$

[illegible]

$$+ 3 m_1 m_2 r_1^{3} r_2 (x_1 - x_3)(x_2 - x_1) + 3 m_1 m_3 x_1 r_3^{3} (x_1 - x_1) \ldots +$$

$$+ 3 m_2 m_3 r_2 r_3 \left[r_3^{2} (x_1 x_2 - x_2 x_3 \ldots) + r_2^{2} (\ldots) \right]$$

[illegible]



$$d_1^2 \frac{d\varphi_1}{dt} = \text{const.} = c_1$$

$$d_2^2 \frac{d\varphi_2}{dt} = \text{const.} = c_2$$

$$d_3^2 \frac{d\varphi_3}{dt} = \text{const.} = c_3$$

[illegible]

$\eta = \frac{[illegible]}{m_1 + [illegible]}$

[illegible] :

$x_3 = [illegible]$ $\qquad \xi = [illegible]$

$\eta = [illegible]$

$y_3 = \frac{[illegible]\,\eta - m_1 \xi - [illegible]}{m_3}$

$\frac{dx_3}{dt} = \frac{[illegible] - m_1 [illegible] - [illegible]}{m_3}$

$\frac{dy_3}{dt} = \frac{[illegible] - m_1 [illegible] - [illegible]}{m_3}$

$v_3 = \sqrt{[illegible]} = \frac{1}{m_3}\sqrt{[illegible]}$

$\frac{dx_1}{dt} = [illegible] = \frac{ds_1}{dt}\,\frac{dx_1}{ds_1} = v_1 \frac{dx_1}{ds_1} = \frac{v_1}{\sqrt{1 - [illegible]}}$

statt ξ [illegible] η [illegible] ζ [illegible]

[illegible]

$\frac{d\xi}{dt} = \frac{1}{M}\left[m_1 \frac{dx_1}{dt} + m_2 \frac{dx_2}{dt} + m_3 \frac{dx_3}{dt}\right]$

[illegible]

$$\xi = \frac{m_1 x_1 [illegible]}{m_1 [illegible]}$$

$$= \frac{[illegible]}{m_1 [illegible]}$$

$$\xi = f[x_1, x_2, x_3, y_1, \ldots]$$

$$\frac{d\xi}{dt} = \frac{\partial \xi}{\partial x_1}\frac{dx_1}{dt} + \frac{\partial \xi}{\partial x_2}\frac{dx_2}{dt} + \frac{\partial \xi}{\partial x_3}\frac{dx_3}{dt} + \frac{\partial \xi}{\partial y_1}\frac{dy_1}{dt} + \ldots$$

$$\xi = \frac{m_1 x_1 [(x_2-x_3)^2 + (y_2-y_3)^2]^{\frac{3}{2}} + m_2 x_2 [(x_1-x_3)^2 + (y_1-y_3)^2]^{\frac{3}{2}} + m_3 x_3 [(x_1-x_2)^2 + (y_1-y_2)^2]^{\frac{3}{2}}}{m_1 [(x_2-x_3)^2 + (y_2-y_3)^2]^{\frac{3}{2}} + m_2 [(x_1-x_3)^2 + (y_1-y_3)^2]^{\frac{3}{2}} + m_3 [(x_1-x_2)^2 + (y_1-y_2)^2]^{\frac{3}{2}}}$$

$$\frac{\partial \xi}{\partial x_1} = \frac{1}{[m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3]^2}\Big\{[m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3][m_1 r_1^3 +$$

$$+ 3 m_2 x_2 (x_1 - x_3) r_2 + 3 m_3 x_3 (x_1 - x_2) r_3] - [m_1 x_1 r_1^3 + m_2 x_2 r_2^3 + m_3 x_3 r_3^3]$$

$$[3 m_2 (x_1 - x_3) r_2 + 3 m_3 (x_1 - x_2) r_3]\Big\}$$

$$= \frac{m_1 r_1^3 + 3 m_2 x_2 r_2 (x_1 - x_3) + 3 m_3 x_3 r_3 (x_1 - x_2)}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3} -$$

$$- \frac{3\xi [m_2 r_2 (x_1 - x_3) + m_3 r_3 (x_1 - x_2)]}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$= \frac{m_1 r_1^3 + 3 m_2 r_2 (x_1 - x_3)(x_2 - \xi) + 3 m_3 r_3 (x_1 - x_2)(x_3 - \xi)}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

[illegible]

$= [illegible] \{ [illegible] + m_2^2 x_1 [illegible] +$

$+ [illegible] + m_1 m_2 r_2^3 r_3 [illegible]$

$+ [illegible] - m_1 m_2 x_1 [illegible] - \cancel{m_2^2 x_1 [illegible]} -$

$- m_1 m_2 x_1 [illegible] - m_1 m_3 x [illegible] + m_1 m_2 [illegible]$

$- m_3^2 x_3 r_3 [illegible]]$

$= \frac{1}{[illegible]^2} \{ [m_1 [illegible] (x_2 - x_1) [illegible] + m_1 m_3 [illegible]$

$+ [illegible] (x_1 - x [illegible]) + m_1 m_2 r_2 [illegible] (x_2 - x_3) [illegible] +$

$+ m_1 [illegible] r_3 (x_3 - x [illegible]) (y_1 - y_1)] \}$

[illegible]

$= 3 \, m_2 x_2 r_3 [illegible] + m_3 x_3 [illegible] - \{ [m_1 r_2 [illegible]$

$m_1 [illegible]$

$$\boxed{\frac{\partial \xi}{\partial x_1} + \frac{\partial \xi}{\partial x_2} + \frac{\partial \xi}{\partial x_3} = [illegible]}$$

$$\frac{\partial \xi}{\partial y_1} = 3\,\frac{[illegible]}{[illegible]}$$

$$\frac{\partial \xi}{\partial y_2} = 3\,\frac{[illegible]}{[illegible]}$$

$$\frac{\partial \xi}{\partial y_3} = 3\,\frac{m_1 [illegible]}{[illegible]}$$

$$\boxed{\frac{\partial \xi}{\partial y_1} + \frac{\partial \xi}{\partial y_2} + \frac{\partial \xi}{\partial y_3} = 0}$$

$$\frac{\partial \eta}{\partial x_1} = 3\,\frac{m_2 r_2 (y_2 - \eta) [illegible] \; x_1 - y}{m_1 [illegible]}$$

$$\frac{\partial \eta}{\partial x_2} = 3\,\frac{m_2 r_2 (y_2 - \eta) [illegible]}{m_1 r_1 + m_2 r_2 [illegible]}$$

$$\frac{\partial \eta}{\partial x_3} =$$

$$\frac{\partial \eta}{\partial y_1} = \frac{m_1 r^3 + 3 m_2 r_2 (y_1 - \eta)(y_2 - \eta) + [illegible]}{m_1 r_1 + m_2 r_2 + m_3 r_3}$$

$$\frac{\partial \eta}{\partial y_2} =$$

$$\frac{\partial \eta}{\partial y_3} =$$

[illegible] $\frac{d}{dt}$ [illegible] ξ

[illegible] $\xi - x$ [illegible]

$(\xi - x)^2$ [illegible] $y)^2$

$\eta = \frac{\eta - y}{\xi - x}$ [illegible]

$\frac{d\eta}{dt} = $ [illegible] $= \frac{(\xi - x_1)\left(\frac{d\eta}{dt} - \frac{dy_1}{dt}\right) - (\eta - y_1)\left(\frac{d\xi}{dt} - \frac{dx_1}{dt}\right)}{(\xi - x_1)^2}$

[illegible]

$\cos^2 \varphi_1 = \frac{1}{1 + \operatorname{tg}^2 \varphi_1} = \frac{1}{1 + \left(\frac{\eta - y_1}{\xi - x_1}\right)^2} = \frac{(\xi - x_1)^2}{(\xi - x_1)^2 + (\eta - y_1)^2}$

$\frac{d\varphi_1}{dt} = \frac{(\xi - x_1)\left(\frac{d\eta}{dt} - \frac{dy_1}{dt}\right) - (\eta - y_1)\left(\frac{d\xi}{dt} - \frac{dx_1}{dt}\right)}{(\xi - x_1)^2 + (\eta - y_1)^2}$

$r_1^2 \frac{d\varphi_1}{dt} = (\xi - x_1)\left(\frac{d\eta}{dt} - \frac{dy_1}{dt}\right) - (\eta - y_1)\left(\frac{d\xi}{dt} - \frac{dx_1}{dt}\right) = \frac{a_1}{m_1}$

$r_2^2 \frac{d\varphi_2}{dt} = (\xi - x_2)\left(\frac{d\eta}{dt} - \frac{dy_2}{dt}\right) - (\eta - y_2)\left(\frac{d\xi}{dt} - \frac{dx_2}{dt}\right) = \frac{a_2}{m_2}$

$r_3^2 \frac{d\varphi_3}{dt} = (\xi - x_3)\left(\frac{d\eta}{dt} - \frac{dy_3}{dt}\right) - (\eta - y_3)\left(\frac{d\xi}{dt} - \frac{dx_3}{dt}\right) = \frac{a_3}{m_3}$

$a_1 = m_1\left[\xi \frac{d\eta}{dt} - \xi \frac{dy_1}{dt} - x_1 \frac{d\eta}{dt} + x_1 \frac{dy_1}{dt} - y_1 \frac{d\xi}{dt} + y_1 \frac{dx_1}{dt} - \eta \frac{d\xi}{dt} - \eta \frac{dx_1}{dt}\right]$

$a_2 = m_2\left[\xi \frac{d\eta}{dt} - \xi \frac{dy_2}{dt} - x_2 \frac{d\eta}{dt} + x_2 \frac{dy_2}{dt} - y_2 \frac{d\xi}{dt} + y_2 \frac{dx_2}{dt} - \eta \frac{d\xi}{dt} - \eta \frac{dx_2}{dt}\right]$

$a_3 = m_3\left[\xi\right.$

[illegible]

$a_1 + a_2 + a_3 = \xi M \frac{d\eta}{dt} - M \xi \frac{d}{dt}$ [illegible]

$a_1 + a_2 + a_3 =$ [illegible]

$+ M$ [illegible]

$+$ [illegible] $M \frac{d}{dt}$ [illegible]

$=$ [illegible] $\frac{d\eta}{dt} - \eta$ [illegible] $\frac{d}{dt}$ [illegible] $- M$ [illegible]

wenn man [illegible]

[illegible]

[illegible]:

$\frac{d}{dt} = 0 \quad \frac{d}{dt} = $ [illegible] $\qquad \xi - \mathfrak{x} = $ [illegible]

$\mathfrak{x} =$ [illegible] $\qquad = p$

$\eta - \mathfrak{y} = q$

$\frac{dp}{dt} = \frac{d\xi}{dt} \qquad \frac{dq}{dt} = \frac{d\eta}{dt}$

$a_1 + a_2 + a_3 = M\left[q \frac{dp}{dt} - p \frac{dq}{dt}\right] + m_1\left[\ldots - x_1 \ldots\right] +$

$+ m_2\left[\ldots - x_2 \ldots\right] + \ldots\left[\ldots\right]$

u. so sind d. [illegible] vom [illegible]

[illegible] $\times M$

mehr [illegible]

[illegible]

[illegible]

[illegible]

$\ldots = \text{constant} = F$

$= B_1$

$B_1 v_1 = B_2 v_2 = \ldots$

$v_1 : v_2 = B_2 : B_1$

$v_1 : v_3 = B_3 : B_1$

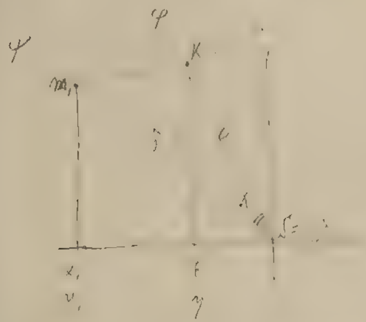
$v_2 : v_3 = \ldots$

$v_1 : v_2 : v_3 = \frac{1}{B_1} : \frac{1}{B_2} : \frac{1}{B_3}$

[illegible]

[illegible]

[illegible]



$\xi = $ [illegible]

$\eta = $ [illegible]

$x = $ [illegible]

$y = $ [illegible]

$\xi \frac{d}{dt} - $ [illegible] $= $

$- \rho \cos\varphi \left[\frac{d\sigma}{dt} \sin\varphi + \right.$ [illegible] $\left. \frac{d\varphi}{dt} \right] - $ [illegible]

$- $ [illegible] $\left[\frac{d\rho}{dt} \right.$ [illegible] $\left. \right] - \rho \sin\varphi$ [illegible]

$= \frac{d\sigma}{dt} \Big[$ [illegible] $\Big] - \frac{d\psi}{dt} \Big[$ [illegible] $\Big]$

$+ \frac{d\varphi}{dt} \Big[$ [illegible] $\Big] + $ [illegible] $=$

$= - \frac{d\sigma}{dt} \sin$ [illegible] $+ \frac{d\psi}{dt}$ [illegible] $\cos\chi + $ [illegible] $=$

$=$

47

$u = \frac{ds}{dt}$ [illegible]

[illegible]

$p = m$ [illegible] $\qquad$ [illegible] $= \int_{t_0}^{t}$ [illegible] dt

$$m\frac{u^2}{2} - m\frac{u_0^2}{2} = \int_{s_0} p\,ds$$

[illegible]

$\frac{dx}{dt} = \frac{ds}{dt}\cos\alpha =$ [illegible] | $\frac{dy}{dt} = \frac{ds}{dt}\cos$ [illegible]

$\frac{d^2x}{dt^2} =$ [illegible] | $\frac{d^2y}{dt^2} =$ [illegible]

$m\frac{d^2s}{dt^2} = m\frac{du}{dt}$ [illegible]

$x =$ [illegible]

$y = r\sin\varphi$

$\frac{dx}{dt} = \frac{dr}{dt}\cos\varphi - r\sin\varphi\frac{d\varphi}{dt}$

$\frac{dy}{dt} = \frac{dr}{dt}\sin\varphi + r\cos\varphi\frac{d\varphi}{dt}$

[illegible] $= \frac{dr}{dt}$

$\perp$ [illegible] $\frac{d\varphi}{dt}$

[illegible] $= \frac{ds^2}{dt^2} = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2$

$K =$ [illegible]

[illegible]

$U =$ [illegible]

$\eta =$ [illegible]

$\xi - x_1 =$ [illegible]

$d_1 \cos$ [illegible] $=$

$=$ [illegible]

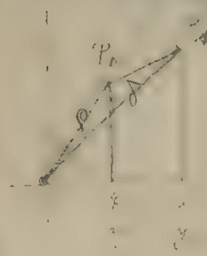
$\eta - y_1 =$ [illegible]

$d_1 \sin$ [illegible]

$d_1^2 [$ [illegible] $] =$ [illegible]

$+2$ [illegible]

$\varphi - \psi = \chi$



$\xi = \rho$ [illegible]

$\eta = \delta \cos \psi$

$x = r \cos \psi$

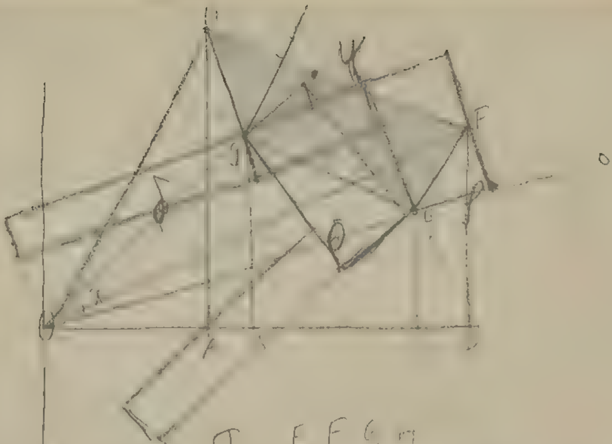
$y = r \sin \psi$

[illegible]

$$= \frac{d\delta}{dt}\left[\ldots\right] + \frac{d\chi}{dt}\left[\ldots\right] + \frac{d\varphi}{dt}\left[\ldots\right] + \frac{d\psi}{dt}\left[\ldots\right]$$

$$= \frac{d\delta}{dt} \sin\chi \cdot \rho + \frac{d\chi}{dt} \cdot \rho \cdot \delta \cos\chi + \frac{d\varphi}{dt} \cdot \rho \cdot \delta \sin\chi$$

$$+ \frac{d\rho}{dt} \sin\chi \cdot \delta$$



$\mathcal{F} = EFGH$

$= ABD + CDF - ADEH$

$= \xi_1 \frac{d\eta}{dt} - \ldots$ $\quad - \ldots - ACEH - CDEI$

$= \frac{1}{2}\left(x_1 - \frac{dx}{dt}\right)\left(y_1 - \frac{dy}{dt}\right)$

$= \frac{1}{2}\left[\xi + \frac{d\xi}{dt} - x_1 - \frac{dx}{dt}\right]\left[y + \ldots + y_1 + \frac{dy}{dt}\right]$

$- \frac{dx}{dt}\left(y_1 + y + \frac{dy}{dt}\right)\frac{1}{2} - \frac{1}{2}\ldots\left(y_1 + y\right) - \frac{1}{2}$

$- \frac{1}{2}\frac{d\xi}{dt}\left(y + y + \frac{dy}{dt}\right)$

$= \frac{1}{2}\ldots$

$- x_1 \frac{dy}{dt} - \ldots$

$+ \xi \frac{dy}{dt} + \ldots - x\frac{dy}{dt} - \ldots - 2y\frac{d\xi}{dt} - \ldots$

$- \ldots + \ldots - \xi y + xy \ldots$

$$m_1\sqrt{\frac{ds_1}{dt}} - C_1 = m_2\sqrt{\frac{ds_2}{dt}} - C_2$$

$$m_1 v_1 \frac{dv_1}{dt} = m_2 v_2 \frac{dv_2}{dt}$$

$$\frac{dv}{dt}$$

$$v_1 B_1 = v_2 B_2$$

$$v_1 : v_2 = B_2 : B_1$$

$\xi - x_1 =$ [illegible]

$= \frac{1}{2}[\xi$ [illegible] η [illegible]

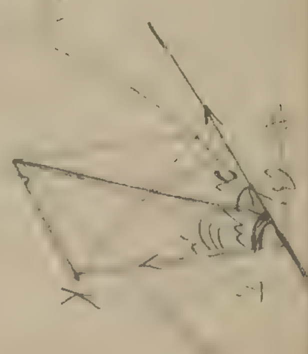
[illegible]

$- x_1$ [illegible]

[illegible]

$= $ [illegible] $+ x_1^2 \frac{d}{dt}$ [illegible]

$= \frac{v_1^2 m}{r}$



[illegible]

$X_1 = \frac{\partial U}{\partial x_1} \qquad Y_1 = \frac{\partial U}{\partial y_1} \qquad U_1 =$ [illegible]

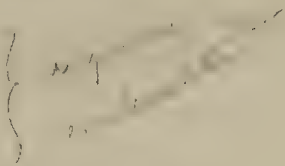
[illegible]

$U_1 = f(x_1, x_2, y_1, y_2 \ldots)$

$m U_1 = x_1 \frac{\partial U_1}{\partial x_1} + x_2 \frac{\partial U_1}{\partial x_2} + y_1 \frac{\partial U_1}{\partial y_1} + y_2 \frac{\partial U_1}{\partial y_2} + \ldots$ [illegible]

[illegible]

$$= \left(\frac{m_1 m_2}{r_{12}} + \frac{m_1 m_3}{r_{13}} + \frac{m_2 m_3}{r_{23}} \right) - 1$$



[illegible]

$$U = \frac{m_1 m_2}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + \ldots}} + \ldots$$

[illegible]:

$$X_1 = \frac{\partial U}{\partial x_1} = m_1 m_2 \frac{x_2 - x_1}{[(x_1 - x_2)^2 + \ldots]^{3/2}} + \ldots$$

$$= \frac{m_1 m_2}{r_{12}^3}(x_2 - x_1) + \frac{m_1 m_3}{r_{13}^3}(x_3 - x_1)$$

$$\frac{m_1}{2} v_1^2 + \frac{m_2}{2} v_2^2 + \ldots = \frac{m_1 m_2}{r_{12}} - \ldots$$

$$\frac{m_1 v_1^2}{2} = a_1 + \frac{m_1 m_2}{r_{12}} + \ldots$$

$$\frac{m_2 v_2^2}{2} = a_2 +$$

$$\frac{m_3 v_3^2}{2} = a_3 +$$

[illegible]

[illegible] v^2 [illegible]

[illegible] u_1^2 [illegible] T'

I' [illegible] $\cos(II'\,\mathfrak{I}) = \dfrac{u_1^2 + d_1'^2 - d_1^2}{2u_1 d_1'}$

[illegible] I' [illegible] $(II'$ [illegible]

[illegible] $\sin(\chi - \varphi)$ [illegible] $II')$ [illegible]

$v_1^2 =$ [illegible]

[illegible] (II') [illegible] $= \sin\left\{\dfrac{u_1^2}{2 \ldots}\right\} + \dfrac{d_1'^2 - d_1^2}{2 \ldots}\Big\} = \dfrac{u_1}{2d_1}$

[illegible] II' [illegible] $\dfrac{\cancel{2u_1^2 d_1'^2} - u_1^4 - d_1'^4 - d_1^4 + 2u_1^2 d_1'^2 + \ldots}{4u_1^2 d_1'^2}$ [illegible]

$= \sqrt{\dfrac{2u_1^2 d_1'^2 + \ldots - u_1^4 - (d_1'^2 - d_1^2)^2}{4u_1^2 d_1'^2}} = \sqrt{1 - \dfrac{u_1^2}{4d_1^2}}$ [illegible]

$v^2 = \ldots^2 + \ldots^2 - 2 \ldots \Big[\sqrt{1 - \dfrac{u_1^2}{4d_1^2}}\,\sin(\chi - \varphi) - \dfrac{u_1}{\ldots}$ [illegible] $\Big]$

[illegible]

[illegible]

[illegible] $=$

$$= -\frac{m_1^2 m_2 m_3}{r_1^3 r_3^3}\left[x_3 y_2 - x_1 y_2 - x_3 y_1 \ldots\right]$$ [illegible]

$$-\frac{m_1 m_2^2 m_3}{r_2^3 r_1^3}\left[x_2 y_3 \ldots\right]$$ [illegible]

$$-\frac{m_1 m_2 m_3^2}{r_1^3 r_2^3}\left[x_3 y_2 - x_1 y_2 - x_3 y_2 + x_1 y_2 \ldots\right]$$ [illegible]

$$= -m_1 m_2 m_3 \left[\qquad\right]\left[\frac{m_1}{r_2^3 r_3^3} + \frac{m_2}{r_1^3 r_3^3} + \frac{m_3}{r_1^3 r_2^3}\right]$$

$$\xi_{1,2} = \frac{\dfrac{m_1 x_1}{(r_2 r_3)^3} + \dfrac{m_2 x_2}{(r_1 r_3)^3} + \dfrac{m_3 x_3}{(r_1 r_2)^3}}{\dfrac{m_1}{(r_2 r_3)^3} + \dfrac{m_2}{(r_1 r_3)^3} + \dfrac{m_3}{(r_1 r_2)^3}} = \frac{m_1 x_1 r_1^3 + m_2 x_2 \cancel{r}_2^3 + m_3 x_3 r_3^3}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$\eta_{1,2} = \frac{Y_1}{X_1}\left[\xi - x_1\right] + y_1 = \frac{Y_1}{X_1}\,\frac{y_2 \ldots X_2 Y_1 + X_1 Y_2 x_2 - X_2 Y_2 x_1 - Y_1 X_1 x_1 + \cancel{Y_1 X_1 x_1}}{Y_2 X_1 - Y_1 X_2} + y_1 =$$

$$= \frac{y_1 X_2 Y_1^2 - y_2 X_2 Y_1^2 + X_1 Y_1 Y_2 x_2 - X_1 Y_1 Y_2 x_1 + y_1 X_1^2 Y_2 - y_1 X_1 X_2 Y_1}{X_1 (Y_2 X_1 - Y_1 X_2)}.$$

$\xi = $ [illegible]

$= \frac{(y_3 - y_1)(\xi - x_1) [illegible]}{[illegible] (\xi - x_3) - (y_1 [illegible]) (\xi - x_1)}$

[illegible]

[illegible] $+ y_1 x_1 \xi_3$ [illegible] $+ y_3 x_1 x_3 - y_1 x_1 x_3$

[illegible]

$\xi_1 = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$ $\quad \xi_3 = \frac{x_2 m_2 + x_3 m_3}{m_2 + m_3}$

$= \frac{(x_1 m_1 + x_2 m_2)(y_2 m_2 + y_3 m_3)}{(m_1 + m_2)(m_2 + m_3)} \cdot x_1 - $ [illegible] $x_3 + \frac{y_1 m_1 + y_2 m_2}{m_1 + m_2} x_1 x_3$

$- \frac{y_2 m_2 + y_3 m_3}{m_2 + m_3}$ [illegible] $+ (y_3 - y_1)$ [illegible]

$x_1^2 y_2$ [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$- y_3 x_1^2 m_1 m_2 - $ [illegible] $- y_3 x_1 x_2$ [illegible]

$= x_1^2 m_1 [y_2 - y_3] + x_1 x_2 [-y_1$ [illegible] $] + x_1 x_3 [y_1 m_1$ [illegible]

$- y_3$ [illegible] $] - x_2^2 [$ [illegible] $] + x_2 x_3 [$ [illegible] $] + x_3^2 [$ [illegible]

d

$K_1 =$

K_1

$= x_1^2 m_1 [\ldots] + x_1 x_2 [m_1(-y_1 + y_2) + m_2(\ldots)] + \ldots$ [illegible]

$+ x_2^2 m_2 [\ldots] + x_2 x_3 [m_2(y_1 + y_2) + m_3(-y_1 \ldots)] + x_3^2 m_3 [y_1 \ldots]$ [illegible]

$$\left[\frac{y_2 m_2 + y_3 m_3}{m_2 + m_3} - y_1\right]\left[\frac{x_1 m_1 + x_2 m_2}{m_1 + m_2} - x_3\right] - \left[\frac{y_1 m_1 + y_2 m_2}{m_1 + m_2} - y_3\right]\left[\frac{x_2 m_2 + x_3 m_3}{m_2 + m_3} - x_1\right]$$

$$[y_2 m_2 + y_3 m_3 - y_1 m_2 - y_1 m_3][x_1 m_1 + x_2 m_2 - x_3 m_1 - x_3 m_2] - [y_1 m_1 + y_2 m_2 - y_3 m_1 - y_3 m_2][x_2 m_2 + x_3 m_3 - x_1 m_2 - x_1 m_3]$$

$= y_2 m_2 x_1 m_1 + \cancel{y_3 m_3 x_1 m_1} - \cancel{y_1 m_2 x_1 m_1} - \ldots$ [illegible]

$- y_2 m_2 x_3 m_1 - \cancel{y_3 m_3 x_3 m_1} + y_1 m_2 x_3 m_1 + \ldots - y_2 m_2 x_3 m_2 - \ldots + y_1 m_2 x_3 m_2 + \ldots$ [illegible]

$- y_1 m_1 x_1 m_2 - \cancel{y_1 m_1 x_3 m_3} + y_3 m_1 x_1 m_2 + y_3 m_2 x_1 m_2 - \cancel{y_1 m_1 x_3 m_3} - y_2 m_2 x_3 m_3 + \ldots$ [illegible]

$+ \cancel{\ldots} + y_2 m_2 x_1 m_3 - y_3 m_1 x_1 m_3 + y_1 m_1 x_1 m_3 + \ldots$ [illegible]

$$= x_1\left[y_2 m_1 + y_2 m_2 - y_3 m_1 - y_3 m_2 + y_2 m_3 - y_3 m_3\right] = x_1\left[m_1(y_2 - y_3) + m_2(y_2 - y_3) + m_3(y_2 - y_3)\right]$$

$$+ x_2\left[y_3 m_2 - y_1 m_2 - y_1 m_3 - y_1 m_1 + y_3 m_1 + y_3 m_3\right] + x_2\left[m_1(-y_1 + y_3) + m_2(-y_1 + y_3) + m_3(-y_1 + y_3)\right]$$

$$+ x_3\left[-y_2 m_1 + y_1 m_1 - y_2 m_2 + y_1 m_2 + y_1 m_3 - y_2 m_3\right] + x_3\left[m_1(y_1 - y_2) + m_2(y_1 - y_2) + m_3(y_1 - y_2)\right]$$

$$= [m_1 + m_2 + m_3]\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$(y_1 - y_2)[m_1 x_1 x_3 \qquad] + (y_3 - y_1)[x_1 x_2 m_1 + x_2^2 m_2 + x_2 x_3 m_3] +$

$+ (y_2 - y_3)[x_1^2 m_1 + x_1 x_2 m_2 + x_1 x_3 m_3]$

$= (y_1 - y_2) x_3 [m_1 x_1 \qquad] + (y_3 - y_1) x_2 [x_1 m_1 + x_2 m_2 + x_3 m_3] + (y_2 - y_3) x_1 [$

$$\boxed{\xi = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}}$$

!

$$\delta_1 = \sqrt{(\xi - x_1)^2 + (\eta - y_1)^2} = \sqrt{\frac{[m_2 (x_2 - x_1) + m_3 (x_3 - x_1)]^2 + [m_2 (y_2 - y_1) + m_3 (y_3 - y_1)]^2}{m_1 + m_2 + m_3}}$$

$$\xi - x_1 = \frac{m_2 (x_2 - x_1) + m_3 (x_3 - x_1)}{m_1 + m_2 + m_3}$$

$$D = \sqrt{(\xi - [illegible])^2 + [illegible]}$$

$$\frac{m_1 x_1 r_1^3 + m_2 x_2 r_2^3 + m_3 x_3 r_3^3}{[illegible]} - \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{[illegible]}$$

$m_1^2 x_1 r_1^3 + \cancel{m_1 m_2 x_2 r_2^3} + m_1 m_3 x_3 r_3^3 + m_1 m_2 x_1 [illegible]$

$+ m_2 m_3 x_2 r_2^3 + [illegible] - [illegible]$

$- m_2 m_3 x_3 r_2^3 - m_1 m_3 [illegible] x - [illegible]$

$= r_1^3 [m_1 m_2 x_1 + m_1 m_3 x_1 - [illegible]]$

$+ r_2^3 [m_1 m_3 x_2 - m_1 m_3 x_3] + r_3^3 [illegible]$

$= r_1^3 m_1 [m_2 (x_1 - x_2) + m_3 (x_1 - x_3)] + r_2^3 m_2 [m_1 x_2 - m_3 x_3] +$

$+ r_3^3 m_3 [m_1 (x_3 - x_1) + m_2 (x_3 - x_2)]$

$= r_1^3 [\cancel{m_1} m_2 x_1 + \cancel{m_1} m_3 x_1 - \cancel{m_1} m_2 x_2 - \cancel{m_1} m_3 x_3] m_1 \qquad m_1 r_1^3 [m_2 (x_1 - x_2) + m_3 (x_1 - x_3)]$

$+ r_2^3 [m_1 \cancel{m_2} x_2 + \cancel{m_2} m_3 x_2 - m_1 \cancel{m_2} x_1 - \cancel{m_2} m_3 x_3] m_2 \qquad + m_2 r_2^3 [m_1 (x_2 - x_1) + m_3 [illegible]]$

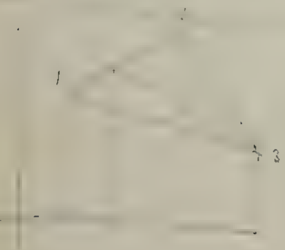
$+ r_3^3 [m_1 \cancel{m_3} x_3 + m_2 \cancel{m_3} x_3 - m_1 \cancel{m_3} x_1 - m_2 \cancel{m_3} x_2] m_3 \qquad + m_3 r_3^3 [m_1 (x_3 - x_1) + [illegible]]$

$$K_1 = \sqrt{X_1^2 + Y_1^2} = \sqrt{\left[\frac{m_1 m_2}{r_3^3}(x_2 - x_1) + \frac{m_1 m_3}{r_2^3}(x_3 - x_1)\right]^2 + \left[\frac{m_1 m_2}{r_3^3}(y_2 - y_1) + \dots\right]^2}$$

$$= \frac{m_1}{r_3^3 r_2^3}\sqrt{\left[m_2 r_2^3 (x_2 - x_1) + \dots\right]^2 + \left[\dots\right]^2}$$

$$K_1 = \frac{m_1\, d_1\,\left[m_1 r_1^3 - m_2 r_2^3 - m_3 r_3^3\right]}{r_2^3\, r_3^3}$$

$r_1^2 = d_1^2 + d_3^2 - 2 d_1 d_3 \cos\alpha_1$



$(x_2 - \xi_3) : (\xi_3 - x_1) = m_1 : m_2$

$x_2 m_2 - \xi_3 m_2 = \xi_3 m_1 - x_1 m_1 \qquad \xi_3 = \dfrac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$

$\eta - y_3 = \dfrac{\eta_3 - y_3}{\xi_3 - x_3}(\xi - x_3)$

$\dfrac{\eta_3 - y_3}{\xi_3 - x_3} = \dfrac{(y_1 - y_3) m_1 + (y_2 - y_3) m_2}{(x_1 - x_3) m_1 + (x_2 - x_3) m_2}$

$\eta - y_1 = \dfrac{\eta_1 - y_1}{\xi_1 - x_1}(\xi - \xi_1)$

$\eta_3 - \eta_1 = \dfrac{\eta_1 - y_1}{\xi_1 - x_1}(\xi - \xi_1) - \dfrac{\eta_3 - y_3}{\xi_3 - x_3}(\xi - \xi_3)$

$$\xi = \frac{\eta_3 - \eta_1 + \frac{\eta_1 - y_1}{\xi_1 - x_1}\xi_1 - \frac{\eta_3 - y_3}{\xi_3 - x_3}\xi_3}{\frac{\eta_1 - y_1}{\xi_1 - x_1} - \frac{\eta_3 - y_3}{\xi_3 - x_3}} = \frac{(\eta_3 - \eta_1)(\xi_1 - x_1)(\xi_3 - x_3) + \xi_1(\eta_1 - y_1)(\xi_3 - x_3) - \xi_3(\eta_3 - y_3)(\xi_1 - x_1)}{(\eta_1 - y_1)(\xi_3 - x_3) - (\eta_3 - y_3)(\xi_1 - x_1)}$$

$$y_{?} = \frac{Y_1}{X_1}\left[\xi_1 - x \text{ [illegible]}\right] =$$

$$\xi_1 - x_1 = \frac{\text{[illegible]}}{\text{[illegible]}}$$

$$= \frac{m_2 r_2^3 [x_2 - x_1] + m_3 r_3^3 [x_3 - x_1]}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$\frac{\cancel{m_1} m_2^2 (x_2 - x_1)(y_2 - y_1) r_2^3}{r_3^3} - \frac{\cancel{m_1} m_2 m_3 (x_2 - x_1)(y_3 - y_1) \cancel{r_1^3}}{\cancel{r_1^3}} + \frac{\cancel{m_1} \text{[illegible]}}{\cancel{r_1^3}}$$

$$- \frac{\cancel{m_1} m_3^2 (x_3 - x_1) \text{[illegible]} r_3^3}{r_3^3} + y_1 \left\{ \frac{m_1 m_2 (x_2 - x_1) r_2^3}{r_3^3} + \frac{\cancel{m_1} m_2^2 r_2^3 \text{[illegible]}}{r_3^3} + \frac{\cancel{m_1} m_2 m_3 \text{[illegible]}}{\cancel{r_1^3}} \right.$$

$$\left. + \frac{m_1 m_3 (x_3 - x_1) r_1^3}{r_3^3} + \frac{\cancel{m_1} m_2 m_3 (x_3 - x_1) \cancel{r_1^3}}{\cancel{r_1^3}} + \frac{\cancel{m_1} m_3^2 \text{[illegible]} r_3^3}{r_3^3} \right\}$$

$$X_1 \qquad m_1$$

$$= \frac{m_2^2 r_2^3}{r_1^3} \left[x_2 y_2 - x_1 y_2 - \cancel{x_1 y_1} + \cancel{x_1 y_1} + y_1 x_2 - y_1 x_1\right] + \frac{m_3^2 r_3^3}{r_2^3} \left[\text{[illegible]}\right]$$

$$+ m_2 m_3 \left[-\cancel{x_1 y_1} + \cancel{x_1 y_1} + x_1 y_3 - x_1 y_3 + x_3 y_2 - x_1 y_2 - \cancel{x_3 y_1} + \cancel{x_1 y_1} + \cancel{y_1 x_2} - \cancel{y_1 x_1} + \text{[illegible]}\right]$$

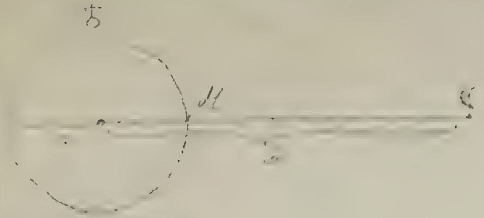
$$+ \frac{m_1 m_2 r_1^3}{r_3^3} y_1 (x_2 - x_1) + \frac{m_1 m_3 r_1^3}{r_2^3} y_1 (x_3 - x_1) : \left[\frac{m_2}{r_3^3}(x_2 - x_1) + \frac{m_3}{r_2^3}(x_3 - x_1)\right]$$

$$r_1^3 r_2^3$$

$$= \frac{m_2^2 r_2^6 y_2 [x_2 - x_1]}{\cancel{r_1^3}} + \frac{m_3^2 r_3^6 y_3 [x_3 - x_1]}{\cancel{r_1^3}} + m_2 m_3 \left[y_3 (x_2 - x_1) + \text{[illegible]} (x_3 - x_1)\right] +$$

$$+ m_1 m_2 r_1^3 r_2^3 y_1 (x_2 - x_1) + m_1 m_3 r_1^3 r_3^3 y_1 (x_3 - x_1) / : \frac{\left[m_2 r_2^3 (x_2 - x_1) + m_3 r_3^3 (x_3 - x_1)\right]}{r_3^3 r_2^3} \left[\text{[illegible]}\right]$$

$$= \frac{m_1 r_1^3 y_1 + m_2 r_2^3 y_2 + m_3 r_3^3 y_3}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$



$K = \frac{m \cdot M}{r^2}$ $\qquad R = 30'27782$

[illegible]

[illegible] : $M = \frac{1}{R^2}$: $\frac{1}{(R \cdot n)^2}$ = [illegible]

$\neq 1 - \frac{2n}{R} = 1 - \frac{2}{[illegible]}$

1: 30'11 = 0033[illegible]

1: 0'96682

1: 0'967 in mittleren [illegible]

1: 0'033

im [illegible] R - [illegible]51 . 60'278

30 1590

2 1 1 1 2

[illegible]

51 [illegible] 221

1: 0'965

1: 0'035 im [illegible]

2. : 56'22 = 0'03510[illegible]

für Sonne : [illegible]

[illegible] $= \frac{1}{00'278^2 \cdot 858^2}$: $\frac{320[illegible] \cdot 74 \cdot 57}{20, [illegible]^2}$

= 4,00 [illegible]

3'51428

3'56032

1'90121

386646

12'84235

12'6[illegible]266

[illegible]

= 1 : 1739

= 0'57516 : 1'00000

17[illegible]79

0'57516

1739

[illegible]

$1 : \frac{1}{(1 - 0'0181)^2} = 1 - 0'0336 : 1$

$= 0'9964 : 1$

[illegible] : [illegible] = 15 [illegible]

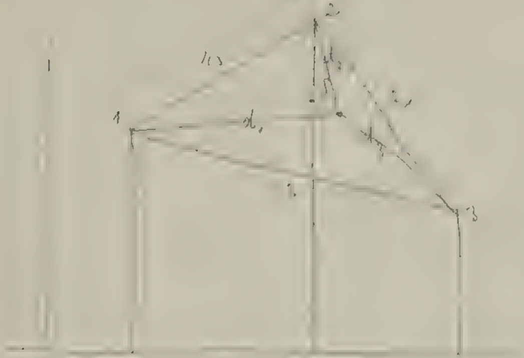
[illegible]

15

[illegible]

1 :

$K = m_1 d_1 [\ldots r_1^3 \ldots r_1^2 \ldots]$ [illegible]



$d_1 \ldots$ [illegible]

$\xi = \dfrac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$

$\eta =$ [illegible]

$\rho_1 = \sqrt{[m_1(x_1 - x_2) + m_3(x_3 - x_2)]^2 + [\ldots]}$ [illegible]

$= \sqrt{\quad}$ [illegible]

$K_1 : K_2 : K_3 = m_1 d_1 r_1^3 : m_2 d_2 r_2^3 : m_3 d_3 r_3^3$

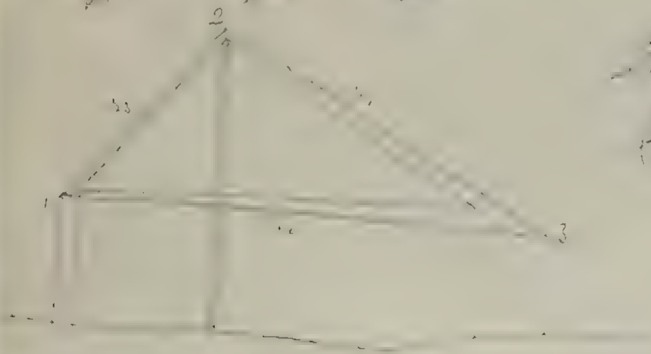
$r_1 = \sqrt{r_2^2 + r_3^2 - 2 r_2 r_3 \ldots} = \sqrt{r_2^2 + r_3^2 - 2 r_2 r_3 \ldots}$

$d_2^2 = d_1^2 + d_3^2 - 2 d_1 d_3 \cos \ldots$ [illegible]

$d_3^2 = d_1^2 + d_2^2 - 2 d_1 d_2 \ldots$

$$m_1\frac{d^2x_1}{dt^2}+m_2\frac{d^2x_2}{dt^2}+m_3\frac{d^2x_3}{dt^2}=0$$

$$m_1\frac{d^2y_1}{dt^2}+m_2\frac{d^2y_2}{dt^2}+m_3\frac{d^2y_3}{dt^2}=0$$



$$\frac{\partial r_3}{\partial x_1}=\frac{x_1-x_2}{r_3}\qquad\frac{\partial r_2}{\partial x_1}$$

$$X_1=\frac{m_1m_2}{r_3^2}\frac{\partial r_3}{\partial(x_2-x_1)}+\frac{m_1m_3}{r_2^2}\frac{\partial r_2}{\partial(x_3-x_1)}$$

$$=\frac{m_1m_2}{r_3^2}\frac{\partial r_3}{\partial x_1}+\frac{m_1m_3}{r_2^2}\frac{\partial r_2}{\partial x_1}$$

$$\frac{m_1}{2}\frac{d v_1^2}{dt}=X_1\frac{dx_1}{dt}+Y_1\frac{dy_1}{dt}$$

$$\frac{d\frac{v_1^2}{2}}{dt}=\frac{\cancel{m_1}m_2}{r_3^2}\frac{\partial r_3}{\partial x_1}\frac{dx_1}{dt}+\frac{\cancel{m_1}m_3}{r_2^2}\frac{\partial r_2}{\partial x_1}\frac{dx_1}{dt}+\frac{\cancel{m_1}m_2}{r_3^2}\frac{\partial r_3}{\partial y_1}\frac{dy_1}{dt}+\frac{\cancel{m_1}m_3}{r_2^2}\frac{\partial r_2}{\partial y_1}\frac{dy_1}{dt}$$

$$=\frac{m_2}{r_3^2}\left[\frac{\partial r_3}{\partial x_1}\frac{dx_1}{dt}+\frac{\partial r_3}{\partial y_1}\frac{dy_1}{dt}\right]+\frac{m_3}{r_2^2}\left[\frac{\partial r_2}{\partial x_1}\frac{dx_1}{dt}+\frac{\partial r_2}{\partial y_1}\frac{dy_1}{dt}\right]$$

$$\frac{1}{2}\frac{d v_1^2}{dt}=\frac{m_2}{r_3^2}\frac{dr_3}{dt}+\frac{m_3}{r_2^2}\frac{dr_2}{dt}\quad(?)$$

2.

1'0

0'5

1'3

1'6

67

49 2.4'2

$5'4^2 = 9'1?$

$7^2 = 49$

$1'7^2 = 3$

$120 : 1'4 = \sqrt{63} = 8$

$160 : 23 = \sqrt{70} = 8'4$

$200 : 2'3 = \sqrt{87} = 9'3$

$240 : 1'4 = \sqrt{176} = 12$

$240 : 2'3 = \sqrt{10?} = 10$

$240 : 2'3 =$

$X_1 X_2 = -$ [illegible]

$- \frac{m_1 m_2 m_3}{\ldots}$ [illegible]

[illegible] $\frac{m_1^2 m_2 m_3}{r_2 r_3}$ [illegible] $- \frac{m_1 m_2 m_3}{\ldots}$ [illegible]

$- \frac{m_2^2 m_3^2}{\ldots}$ [illegible] $\frac{\ldots}{r_1^2}$

$X_1 Y_2 = -\frac{m_1 m_2^2}{r_3^4}$ $(x_1 - x_2)$ [illegible] $+$ [illegible] $- \frac{m_1 m_2 m_3}{\ldots}$ [illegible]

$- \frac{m_1 m_3}{r_1^2 r_2^2}$ $\frac{x_3 - x_1}{\ldots}$ [illegible]

$X_2 Y_1 = -\frac{m_1 m_2^2}{r_3^4}$ [illegible] $+ \frac{m_1 m_2^2 m_3}{\ldots}$ $(x_3 - x_2)$ [illegible]

$-$ [illegible] $(x_3 - x_1)$ [illegible] $\frac{\ldots}{r_1 r_2}$

[illegible] $= - \frac{m_1^2 m_2^2 (x_1 - x_2)}{r_3^6}$ [illegible] $- \frac{m_1^2 m_2 m_3}{\ldots}$ [illegible] $+$ [illegible]

$+ \frac{m_1 m_2 m_3^2}{r_1^3 r_2^3}$ [illegible] $+ \frac{m_1^3 m_2}{\ldots}$ [illegible] $x_2 - m_1 m_2 m_3$

$- \frac{m_1^2 m_2 m_3}{r_1^3 r_3^3}$ [illegible] $- m_1 m_2$ [illegible] $\frac{\ldots}{r_1^3}$ [illegible] $-$ [illegible] $(x_1 - x_2)$ [illegible]

$+ m_1 m_2^2 (x_1 - x_2)$ [illegible] $- m_1 m_2$ [illegible] $- m_1 m_2$ [illegible] $x_3 - x$ [illegible] $\frac{\ldots}{r_2^3 r_3^3}$

$+ \frac{m_1 m_2 m_3}{r_1^3 r_2^3}$ [illegible]

$X_1 = $ [illegible]

$X_2 = $ [illegible]

$X_3 = -$ [illegible] $\frac{[illegible]}{r_3}$ $-$ [illegible]

[illegible]

$\operatorname{tg}\beta = $ [illegible]

$= \frac{2 b \operatorname{ctg}[illegible]}{[1-\operatorname{ctg}^2[illegible]] - b\,[1+\operatorname{ctg}^2[illegible]]}$

$\operatorname{tg}\beta = \frac{\operatorname{ctg}[illegible] - [illegible]}{1 - [illegible]}$

$Y_1 = $ [illegible]

$Y_2 = -\frac{m_2 m_1}{r_1^2}$ [illegible]

$Y_3 = \frac{m_1 m_3}{r_1^2}$ [illegible] $+$ [illegible]

$\operatorname{tg}\tau_1 = \frac{Y_1}{X_1}$ $\quad \operatorname{tg}\tau_2 = \frac{Y_2}{X_2}$ $\quad$ [illegible]

$\eta_1 - y_1 = \frac{Y_1}{X_1}(\xi_1 - x_1)$

$\eta_2 - y_2 = \frac{Y_2}{X_2}(\xi_2 - x_2)$

$y_1 - y_2 = \frac{Y_2}{X_2}[\xi - x_2] - \frac{Y_1}{X_1}[\xi - x_1]$

$(y_1 - y_2) X_2 X_1 = [Y_2 X_1 - Y_1 X_2]\,\xi - Y_2 X_1 x_2 + Y_1 X_2 x_1$

$\xi_{1,2} = \frac{(y_1 - y_2) X_2 X_1 + X_1 Y_2 x_2 - X_2 Y_1 x_1}{Y_2 X_1 - Y_1 X_2}$

$\xi_{2,3} = \frac{(y_2 - y_3) X_3 X_2 - X_2 Y_3 x_3 - X_3 Y_2 x_2}{Y_3 X_2 - Y_2 X_3}$

[illegible]

$$(x_3 - x_1)(x_3 - x_2)(\ldots) - (x_3 - x_1)(\ldots) x_2 + (x_3 - x_2)(\ldots) x_1$$

$$(x_3 - x_1)\left[x_3 \ldots - x_3 y_2 + x_2 y_1 - x_2 y_2 + x_2 y_3\right] - (x_3 y_1 - x_2 y_1 - x_3 \ldots + x_2 y_3) x_1$$

$$- x_3 \left[x_3 y_1 - x_1 y_1 - x_3 \ldots + \ldots\right] + x_1 \left[x_3 [y_2 - y_1]\right]$$

$$= x_3 \left[x_3 y_1 - x_2 y_1 - x_3 y_2 + x_2 y_3 + x_1 (y_2 - \ldots)\right]$$

$$(x_2 - x_1)^2 (\ldots) - (x_2 - x_1)(\ldots) x_2 + (x_2 - x_1)(\ldots) x_1$$

$$(y_1 - y_2)\left[(x_2 - x_1)^2 - x_1 x_2 + x_2^2 + x_1^2 - \ldots\right] = 0$$

$$\xi_{1,2} = \left[x_3 y_1 - x_2 y_1 - x_3 y_2 + x_2 y_3 + \ldots\right]\left[\frac{m_1^2 m_2 m_3 x_1}{r_2^3 r_3^3} + \frac{m_1 m_2^2 m_3 x_2}{r_1^3 r_3^3} + \frac{m_1 m_2 m_3^2 \ldots}{r_1^3 r_2^3}\right]$$

$$= \qquad m_1 m_2 m_3 \left[\frac{m_1 x_1}{r_2 r_3} + \frac{m_2 x_2}{r_1 r_3} + \frac{\ldots}{\ldots}\right]$$

$$\xi_{2,3} = \left[x_1 y_2 - x_3 y_2 - x_1 y_3 + x_3 y_1 + x_2 y_3 - x_2 y_1\right] \qquad \left[\frac{m_2 x_2}{r_3 \ldots} - \right.$$

~~$V = f(r_1, r_2, \ldots)$~~

$$V = f(r_1, r_2, r_3) = f_1(d_1, d_2, d_3)$$

$$V_1 = d_1 \frac{dV_1}{dd_1} + \ldots$$

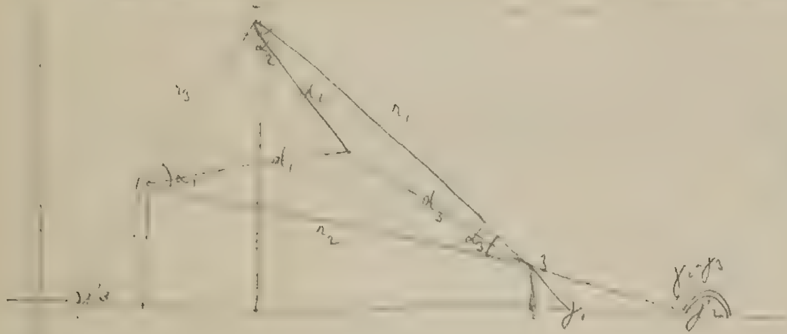
$$= d_1 K_1 + d_2 K_2 + d_3 K_3$$

$$m V_1 = (m_1 d_1^2 r_1^3 + m_2 d_2^2 r_2^3 + m_3 d_3^2 r_3^3) \frac{(\ldots m_2 \ldots)}{r_1 r_2 r_3}$$

$$= m_1 r_1^3 r_2 r_3 [\ldots - m_2 m_3 \ldots] + m_2 r_2^3 r_3 r_1 [m_3 \ldots - m_3 m_1 r_3 \ldots] + m_3 r_3^3 r_1 r_2 [\ldots] \Big\} : (m_1 \ldots)(r^3 \ldots)$$

$$= m_1 m_2 \ldots + m_1 m_3 \ldots - m_1 m_2 m_3 \ldots (r_2^2 + \ldots - r_1^2) + m_2 m_3 \ldots - m_4 m_2 m_3 \ldots + m \ldots$$

[illegible]



$$= \sqrt{\left[m_2 r_2^3 \cos\gamma_3 - m_3 r_3^3 \cos\gamma_2\right]^2 + \left[m_2 r_2^3 \sin\gamma_3 - m_3 r_3^3 \sin\gamma_2\right]^2}$$

$$= \sqrt{(r_2 r_3)\{m_2^2 r_2^4 \cos^2\gamma_3 + m_3^2 r_3^4 \cos^2\gamma_2 + 2 m_2 m_3 r_2^2 r_3^2 \cos\gamma_3 \cos\gamma_2 + m_2^2 r_2^4 \sin^2\gamma_3 + 2 m_2 m_3 r_2^2 r_3^2 \sin\gamma_3 \sin\gamma_2 + m_3^2 r_3^4 \sin^2\gamma_2\}}$$

$$= \frac{\sqrt{r_2 r_3 \left[m_2^2 r_2^4 - m_3^2 r_3^4 - 2 m_2 m_3 r_2^2 r_3^2 \cos\alpha_1\right]}}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$r_1^2 = r_2^2 + r_3^2 - 2 r_2 r_3 \cos\alpha_1$$

$$\cos\alpha_1 = \frac{r_2^2 + r_3^2 - r_1^2}{2 r_2 r_3}$$

$$d_1 = \frac{\sqrt{r_2 r_3 \left[m_2^2 r_2^4 + m_3^2 r_3^4 - m_2 m_3 r_2 r_3 (r_2^2 + r_3^2 - r_1^2)\right]}}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$d_1 = \frac{\sqrt{r_2 r_3 \left[(m_2 r_2^2 + m_3 r_3^2)^2 - m_2 m_3 r_2 r_3 [r_2 + r_3]^2 - r_1^2\right]}}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$K_1 = m_1 \frac{\sqrt{r_2 r_3 \left[m_2^2 r_2^4 + m_3^2 r_3^4 - m_2 m_3 r_2 r_3 (r_2^2 + r_3^2 - r_1^2)\right]}}{[illegible]^3 \; r_3^3}$$

7-2

O promieniowaniu i analizie widmowej.

Jedna z najnowszych gałęzi fizyki; zasadn. ~~...~~ odkrycie Kirchhoffa dopiero 1860 i dopiero w ostatnich latach ogromne postępy.

Nawet nie wiadomo do czego to zaliczyć, czy do optyki, czy termiki?

Zwykle analiza widmowa ct. do optyki ale to tylko spec. przyp. ogólnego prom. cieplnego, najściślejszy związek z zasadami termodyn., z drugiej strony związek z elektronami

Dawniejsza (klasyczna) optyka zajmuje się prawidłami rozchodzenia się światła przy czem rodzaj światła uważa już za coś danego. Skąd ono pochodzi to nas obojętne.

W tej ~~...~~ to wszystko uważać będziemy za znane i interesować będzie nas głównie sposób powstawania promieni i zanikania (absorpcyi).

Historyczny rozwój z dwóch punktów widzenia optycznego i kalorycznego

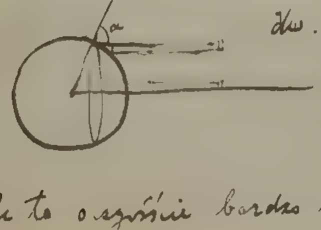
Lambert 1760 (Photometria) znał już prawo $J = \frac{c}{r^2}$

a dla nachylonych: $\frac{c \cos\alpha}{r^2}$

Takie samo prawo wyobrażał on dla wysyłania światła

prawo cosinusowe Lamberta

i powołał się jako dowód że Słońce wydaje się równie jasne we wszystkich częściach



$$d\omega \cdot s = \frac{d\omega}{\cos\alpha} \cdot \sigma$$

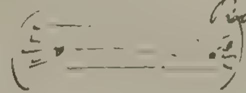
$$\sigma = s \cos\alpha$$

ale to oczywiście bardzo niedokładna ~~...~~ metoda, rzeczywiście to tylko przybliżenie prawdziwe jak późniejsze badania dow. wykazały. (Stąd dowód fałszywy!)

Już Lambert twierdził na podstawie dośw. że promienie ciepła według tych samych praw się rozchodzą jak prom. światła i wykazuje także samo prawo dla ciepła prom.

Leslie przybliż. potwierdza prawo cosin.

Pictet (1790)



nie tylko ciepło ale też zimno

więc czy istnieje osobne promieniowanie zimna?

Przeciwnie zimno tylko = brak ciepła

Zupełnie jasno zrozumiał to Prevost 1809

„ciało w otoczeniu o równej temp. zachowuje się jak jezioro do którego deszcz pada podczas gdy równie dużo wody paruje".

Przy nierównej temp. obserwujemy różnicę prom. w jednym a drugim kierunku.

Dulong i Petit 1817 zależność od temp. ma^t

Jak daleca jednak ściśle uważano prom. światła a ciepła, jako coś różnego

dopiero powoli poznano, że każdy prom. świetlny jest też cieplnym.

Zbadania prace Melloniego ~~1833–1845~~ 1840 1806 i Knoblauch

Absorpcji z kamieni,

Fizeau & Foucault interferencya zwierc. Fresnela, eff. etc. 1847 6 promieni pomiary długości fal cieplnymi

Knoblauch jeszcze obszerniej

Także polaryzacya Melloni Knoblauch i inni, Provostaye & Desains etc.

Skręt płaszcz. polaryzacji Desains

Już tylko interes historyczny

Hist. Wstęp: Lambert
Melloni
Stewart Kirchhoff Bunsen, Clausius
Spectrosk. Lockyer, Plücker, Tyndall, ... Rowland, Deslandres, ~~Langley~~
Teorya prom. ogólna. Langley, Michelson, Wien, Paschen, Lummer, Pringsheim, Planck etc.

Światło = rodzaj prom. cieplnych
(A. Termoluminescencya) (B Inne lumin.
Prawo Kirchhoffa
T.zw. prawo Clausiusa, teorya Smol., prawo Lamberta
Dowody doświadcz. jakościowe (Exp.)

Dwa zadania: Prom. ciała czarnego, Absorpcya indywid.

Metody doświadcz. ↗:
β). Spectrosk.
Ugięcie, siatki dyffr.
Michels. interfer.
Widma normalne, etc. Rowland, Kayser
(Exp.)

β). Oko ludzkie Fotogr..
Termo-
Bolometr
Radiometr Nichols-Rubens
(Exp.)

α). Ciało czarne idealne, realizacya jego (Exp.)
Langley
Paschen, Lummer-Pringsheim

Teorya: Zasada Dopplera, Ciśnienie promien. (Lebedew)
Michelson,
Wien - Planck, rezultat teoret. - prakt.

Absorpcya:

prawo Kirchhoffa etc. ~~W~~ Widma liniowe, iskrowe, etc. (Exp.)
Doppler, Przesunięcie wskutek ciśnienia, Humphreys & Mohler
związek z spółcz. załamania, ~~teorya ...~~

~~...~~ dysp. anormalna, ~~... teorye~~ (Exp.)

teorye, Helmholtz, Drude etc.

Potwierdz. teoryi przez nowsze dośw. w widmie pozaczerwonem

Resultaty co do gazów pierwiastków (Exp.)
Odkrycia nowych pierwiastków: In, Th, .. A, He Ne, Xe, Kr, Rad. etc.
Prawo seryi: Balmer, Rydberg, Kayser & Runge

Zagadka teoret.

~~...~~ widma pasmowe (Exp.)

widma abs. ciał ciekłych i stałych, Jony (Exp.)

Barwy ciał (Exp.)

Praktyczne zastosowania analizy spektr.

Krew, Bessemer Pr. Barwiki anilinowe etc., Andere chem. anwendungen (Exp.)
mokra i spektr.

Astronom.: skład ciał nieb.

prędkości ich

B. Luminescencye inne ciał. Fluorescencya (Exp.)

Phosphorescencya (Exp.)

Redygowanie

Ciśnienie promieniowania

Maxwell

Energia 

$$\sigma = a \sin 2\pi\left(\frac{t}{\tau} - \frac{x}{\lambda}\right)$$

$$\dot\sigma = a\,\frac{2\pi}{\tau} \cos 2\pi\left(\frac{t}{\tau} - \frac{x}{\lambda}\right)$$

$$\int \frac{\dot\sigma^2}{2}\,dx = \frac{4\pi^2 a^2}{\tau^2}\int$$

Promieniowanie spełnia prawa termodynamiki

Odwracalne dopóki równowaga t.j. [illegible] o jednakowej temp.

Newton $\propto (t - t_0)$

Dulong Petit (1817) termometr stygnący w naczyniach

sądzili że przewodn. ciepln. zmienia się przy gęstości gazu (ciśn. zmniejsz. do 2mm)

$S = m a^T$

Stefan (1879)

~~[illegible]~~ Rosetti, Schleiermacher, Paschen (Platyna do $T^0 - 6 \cdot 4$; C; Pt etc.)

Boltzmann: schwarzen Strahler 1884

Wien Lummer 1895

Lummer & Pringsheim 1897

Abs. temp.	wychylenie	Abs. temp. obl.	Δ
373·1	156	374·6	
723·0	3320	724·3	
868	6910	867·1	
1092	16400	1074	
1378	44700	1379	
1535	67800	1531	

Paschen 1899

$$dW = a \frac{ds\, ds'}{r^2} \left(T^4 - T'^4\right)$$

$a = 1{\cdot}71 \cdot 10^{-5}$ ergów

$= 0{\cdot}408 \cdot 10^{-12} \frac{cal}{sec}$

ds, $ds' \perp r$

~~[illegible]~~ $\sum ds'_{n} = r^2 \pi$

$$e = a \pi\, ds \left(T^4 - T'^4\right)$$

Np. $T = 373$, $T' = 273$

$e = 0{\cdot}01763 \frac{gcal}{sec}$

[illegible]

$$0{,}408 \cdot 10^{-12} \frac{ds}{r^2} \left(T^4 - T'^4\right) = \frac{3}{60}$$

[illegible]

$$ds = 2r^2\pi = \left(\tfrac{1}{2}\, 32'\right)^2 \pi : 2\pi$$

$$ds = r^2 \cdot \left(\frac{32'}{2}\right)^2 \pi$$

$$T = 6200^\circ$$

Kirchhoff 1859

emisja jakiegokolwiek ciała $\frac{\text{w danej barwie}}{\text{w danej temp.}}$ = emisja ciała dosk. cz. w tej samej temp. i barwie × absorpcja ciała ...

ważne wytłumaczenie do prom. cieplnego, i dla równowagi cieplnej

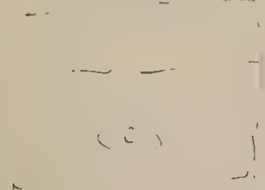
niezależ do fosforescencji, luminescencji, radioaktywności, reakcji świetlnych

Prawo tyle pochłaniają ile emitują

— emisja ten

' = a + r + p + s

dosk. czarne - takie które wszystko absorbują (miejsce, mierz. w środku ...



1). ciało czarne w otoczeniu czarnym

całkowite promieniowanie jego niezal. od rodzaju i kształtu

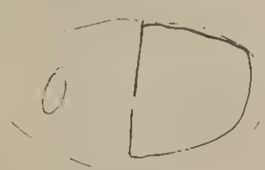
2). ich samo natężenie na barwy (λ, T) i Planckowski wzór

3). ciało nie czarne

4). zależność od zdoln. źródła

---

w otoczeniu



1). promieniowanie wychodzące przez mały otwór = czarne

2). emisja ciała prom. = 0

3). ... = 0

r + s + t + a = 1).

2). $\int (r+s+t) E_0 = E_0(1-a)$

zatem ciało by stale emitowało $E = E_0 a$

3) Strahlung –

1).

$e = \int e_\lambda \, d\lambda$ $\qquad E = \int A_\lambda e_\lambda \, d\lambda = \int A_\lambda e_{1\lambda} \, d\lambda = \cdots$

$e_\lambda = e_{1\lambda} = e_{2\lambda} \cdots$

$e = \int e_\lambda d\lambda$

[illegible] $A = 1$ [illegible]

2).

$e_\lambda = E_\lambda + S$

$S = (1 - A_\lambda) e_\lambda$

$e_\lambda = E_\lambda + (1 - A_\lambda) e_\lambda$

$E_\lambda = A_\lambda e_\lambda$

[illegible]

[illegible]

$$\frac{1}{\lambda} = A - \frac{B}{n^2}$$

Ogilvie $\frac{1}{\lambda} = A - \frac{B}{n^2} - \frac{C}{n^4}$ ~~Rydberg~~, Kayser & Runge
Runge & Paschen

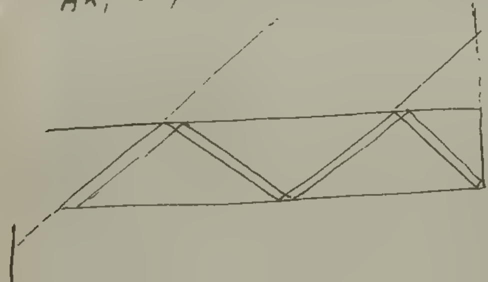
Rydberg $\frac{1}{\lambda} = A + \frac{B}{(m+\mu)^2}$

$= A + \frac{B}{n^2} + \frac{C}{n^3} +$

I Li Na K 3 serie doublets

II Mg Ca Sr, Zn Cd Hg 2 serie triplets

III Al, In, Tl 2 serie [illegible].



$c =$ 50

$$h\nu = 6{,}5 \cdot 10^{-27} \cdot \frac{3 \cdot 10^{10}}{0{,}001} = 6{,}5 \cdot 3 \cdot 10^{-14}$$

$$= 2 \cdot 10^{-13}$$

$$\bar{L} = \frac{3}{2} k\theta = \frac{3}{2} \cdot 1{,}3 \cdot 10^{-16} \cdot 273$$

$$= 5{,}4 \cdot 10^{-14}$$

$$\alpha \int e^{-\beta(\xi^2+\eta^2+\zeta^2)} d\xi\, d\eta\, d\zeta \left[\log \alpha - \beta(\xi^2+\eta^2+\zeta^2)\right]$$

$$= \alpha \log \alpha \cdot \left(\frac{\pi}{\beta}\right)^{3/2} - \cancel{\alpha \left(\frac{\pi}{\beta}\right)^{3/2}}$$

z Rektorats Univ. 13/I 1912 I. 592

Jadwiga Falkowska

XIV 307/5 20/1

8)

Tak trwało to jednak do połowy XVIII w. [illegible] [illegible]

Linie Fraunhofera 1814

[illegible] odkrycie analizy spektr. Kirchhoff i Bunsen 1860 [illegible] [illegible] [illegible] linie o [illegible] [illegible]

prawo Kirchhoffa co do absorpcji — emisyi

Co prawda że [illegible] to już dawniej [illegible] Provostaye & Desains 1853

Balfour Stewart 1857

Ogromny rozwój analizy widm. sam Kirchh. & B. badali [illegible] [illegible] minerały

i odkryli tym sposobem Rb, Cs

później Tl Crookes In Reich Ga Lecoq de Boisbaudran etc. etc.

Huggins, Lockyer Angström [illegible] widma [illegible] Słońca później [illegible]

Rowland 1888 atlas ~~[illegible]~~ widma słon. fotogr. [illegible]

Deslandres [illegible] dokładność 0.000.01 μ

Regularności w widmach: Balmer, Rydberg, Kayser & Runge

[illegible] promien. ciepliw. Stefan 1879 Boltzmann

Klasyczne badania Langleya 1886 bolometr rozkład ciepła w [illegible] [illegible] aż do tego 40°.

Michelson 1887 ~~[illegible]~~ Rayleigh Wien 1890 Planck

Lummer & Pringsheim, Paschen etc.

Tak że obecnie nawet temp. księżyca możemy mierzyć.

Desains 1868	Therm. Słońca	
Langley 1883	28 μ	
1888	28 μ	

			Δ
α	6563·07		
β	4861·57	4861·52	+0·05
γ	4340·53	4340·63	−0·10
δ	4102·00	4101·90	+0·16
ε	3970·33	3970·22	+0·11
ζ	3889·15	3889·20	−0·05

Evershed (22/1 1898)

Angström [illegible] $= 10^{-7}$ mm $= 10^{-10}$ m

=

$$\frac{1}{\lambda_n} = \frac{1}{3647\cdot 2}\left(1 - \frac{4}{n^2}\right)$$

$$= 1\cdot 096750\left[\frac{1}{4} - \frac{1}{n^2}\right] \cdot 10^{-3}\ (\text{Å})$$

$n = 3$ —— 32

$$\nu = N\left(\frac{1}{4} - \frac{1}{n^2}\right)$$

(I Nebenserie) breite Li.

Pickering ζ Puppis 1896

$$\nu = N\left[\frac{1}{4} - \frac{1}{(m+\frac{1}{2})^2}\right]$$

(II Nebenserie) scharfe [illegible] schwachen

Hauptserie Rydberg scharfe starke Linien in U.V.

$$\nu = N\left[\frac{1}{(1\frac{1}{2})^2} - \frac{1}{m^2}\right] \quad m = 2, 3 \cdots$$

Na Wood fluorescence spectrum 48 lines of $D_1 D_2$ resonance previously known only 7 series

Li tak samo 3 serye

Na K Rb tak samo ale podwójne linie $\nu_1 - \nu_2$ = const dla components Hartley & Julius

w Nebenserye

ale nie ...

w II Serye [illegible] tryplety (Hauptserie nie ma)

Runge & Paschen effekt Zeemana ten sam dla serye

Dodane k Rydberg w optical H. parawie by: Ritz p. 170 – 173!

$$\frac{1}{\lambda} = N\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \qquad \text{I Neb.}$$

$$N\left(\frac{1}{n^2} - \frac{1}{(m+\frac{1}{2})^2}\right) \qquad \text{II } N.$$

istotnie Paschen [illegible] 1908:

$\lambda = 18751{\cdot}3$ $\quad \frac{1}{3^2} - \frac{1}{4^2}, \; 18751{\cdot}6$

$12817{\cdot}6$ $\quad \frac{1}{3^2} - \frac{1}{5^2}, \; 12818{\cdot}7$

dla innych:

Rydberg $\quad \frac{1}{\lambda} = A + \frac{B}{(m+\mu)^2}$

K. i R.: $\quad \frac{1}{\lambda} = A' + \frac{B'}{m^2} + \frac{C'}{m^4} + \cdots \qquad m = 2, 3$ ~~[illegible]~~ [illegible]

Ritz: przyjmuje wzór typu:

$$\frac{1}{\lambda} = N\left\{\frac{1}{\left(n+a+\frac{b}{n^2}\right)^2} - \frac{1}{\left(m+a'+\frac{b'}{m^2}\right)^2}\right\}$$

N to samo co dla wodoru

$n = 1\frac{1}{2} \qquad n = 2, 3 \cdots \qquad$ Haupts. $\Big\}$ więc wszystkie linie $n = 1^{1/2}$ $n = 2$!

$n = 2 \qquad m = 2\frac{1}{2} \quad 3\frac{1}{2} \cdots \quad$ II Neben. scharfe Linie

$n = 2 \qquad m = 3 \quad 4 \quad 5 \cdots \quad$ I Neb. diffuse Linie; zmienne a' b' $\Big\}$ takie same $m = 3 \cdots$

obie kończą się $m = \infty$ w tem samem miejscu

Na	1 Nebenserie	2 Nebenserie
n=3 5890.16 / 5896.9 $\Delta \sim \frac{1}{n^4}$	3 8194.76 / 8184.33	
	4 5688.26 / 5682.9	4 6161.5 / 54.62
n=4 3303 / 3302	5 4983.5 / 4979.3	
5 2853	6 4668.95 / 4664.68	
6 2680		
	7	8 4392.99
9 2511.77 / 58	9 4323.94	

$$\lambda = A \frac{n^2}{n^2-4}$$

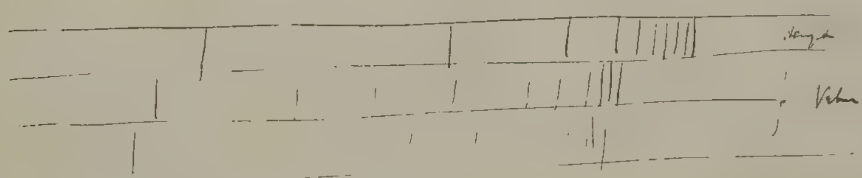
$$\frac{1}{\lambda} = \ldots$$

K. i R.: $$\frac{1}{\lambda} = A - \frac{B}{n^2} - \frac{C}{n^4}$$

Rydberg: $$\frac{1}{\lambda} = A + \frac{B}{(m+\mu)^2}$$

$$= A + \frac{B}{m^2} + \frac{C}{m^3}$$

(B to samo co u wodoru)



0	1	2	3	4	5	6	7	8
He 4	Li 7	Be 9	B 11	C 12	N 14	O 16	F 19	
Ne 20	Na 23	Mg 24	Al 27	Si = 28	P = 31	S = 32	Cl = 35.5	
A 40	K 39	Ca 40	Sc 44	Ti = 48	V = 51	Cr = 52	Mn = 55	Fe Ni Co
	Cu = 64	Zn 65	Ga 70	Ge = 72	As = 75	Se = 79	Br = 80	
						Mo = 96		Ru
Kr 82	Rb 85.4	Sr 87.6	Y 89	Zr = 90.6	Nb = 94			
	Ag = 108	Cd 112	In 114	Sn = 118.5	Sb = 120	Te = 127	J = 127	
X 128	Cs = 133	Ba = 137	La 138	Ce = 140	—	—		
	—	--	—	—	—	—		Os Ir Pt
	—	--	Yb = 173	—	Ta = 183	W = 184		
	Au = 197	Hg = 200	Tl = 204	Pb = 207	Bi = 208	—		
		Ra 225		Th = 232	—	U = 240		

Humphreys & Mohler (1896)   15 atm. linie delik. w skrzyni zielonej

na 10 Atm przesunięcie linij żółtej $\frac{23}{1000}$ ÅE

5896.

ÅE

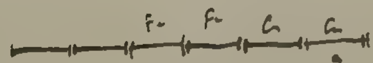
$m\frac{d^2x}{dt^2} = \frac{eH}{m}\frac{dy}{dt}$ $x = A\cos\alpha t$ $-\alpha^2 A = -\alpha B c$ $\alpha = \frac{B}{A}c = \frac{A}{B}c$

$\frac{d^2y}{dt^2} = -\frac{eH}{m}\frac{dx}{dt}$ $y = B\sin\alpha t$ $-\alpha^2 B = -\alpha A c$ $A = B$

$\alpha = c$

Ritz

$\Delta = \frac{1}{\lambda_1} - \frac{1}{\lambda_2}$ $\nu = \frac{eH}{m}$



p. 176

Hartley 1883 $\Delta\nu$ = [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible] [illegible] [illegible] [illegible] $\sim a^2$ 1/3 R.

Zn 64·9 Cd 111·83

a	63·64
b	6[illegible]
c	48
d	17
e	64·[illegible]3
f	65·28
	15
	12
	16
	64·6[illegible]
	64·77

Sn Pb As Sb Cr Ru [illegible]

$\nu = \frac{1}{\lambda}$

Sn:

3801·16	3175·13	−0·01
3330·71	2840·05	+0·01
2850·72	2483·[illegible]	+0·01

$$\frac{1}{3175{\cdot}13 \cdot 10^{8}} = \frac{1}{3801{\cdot}16 \cdot 10^{8}} + 5187{\cdot}03$$

38[illegible]0·7[illegible] [illegible]8/10[illegible] [illegible]/10[illegible] 0/5

Bandenspektren:

Kopf

Gruppe

$$\frac{1}{\lambda} = A + b n^2$$

Stickstoff Kopf 3914·6 63 Linien $\frac{1}{\lambda} = 255{\cdot}454 + 0{\cdot}0015335(n-1)^2$

Gasbande 160 Linien

Cathodenbanden Deslandres: $\frac{1}{\lambda} = A n^2 + B n^2 + C$

[illegible] $\Delta \sim \lambda$. Haupt : I N. : II N. = 1:2:4

N^{10} Kostek = molekuł na każdej P liczb

P komórek

1 2 3 4 - 10 kostka

oczek ~~3 6 1 1 1 6 6 1 1 4~~

2 4 3 1 3 5 4 3 5 2

1	1
2	2
3	3
4	2
5	2
6	0

2 3 5

1 2 | 3 4 5 | 6 7 8 9 10

1 2 |

Rozkład liczb

1 1 1 1 1 1 1 - - 1 od największy 1

inny

1 2 1 1 - 1

2 1 1 -

1 1 2

$$\frac{(\alpha+\beta+\gamma)!}{\alpha!\beta!\gamma!}$$

~~Permutacje~~ względna liczba permutacji

N Rezonatorów = 3

P Elementów = 4

~~$\frac{8!}{2!\,4!} = 7.4$~~ $\frac{6!}{2!\,4!}$

Rezonatory rozróżnialne ale elementy nie

112333

~~111111~~

~~111112~~

~~111113~~

~~111122~~

~~111133~~

~~111123~~

0 0 4	0 1 3	1 1 2
0 4 0	0 3 1	1 2 1
4 0 0	1 0 3	2 1 1
	1 3 0	
	3 0 1	
	3 1 0	
	0 2 2	
	2 0 2	
	2 2 0	

ogólnie $\frac{(N+P-1)!}{(N-1)!\,P!}$ = liczba kombinacyj z powtórzeniem N elementów P klasy

$$\frac{N(N+1)\cdots(N+P-1)}{1.2\ldots P}$$

$N=2$ || $P=4$ || 04 13 22 / 40 31 $\quad \frac{5!}{1!\,4!} = 5$

0 a b c d $\qquad$ 0 a b c d

$P = \frac{UN}{\varepsilon}$

$$S_N = k \lg \frac{(N+P)!}{N!\,P!}$$

$$= k\left\{(N+P)\lg(N+P) - N\lg N - P\lg P\right\}$$

$$S = \frac{S_N}{N} = k\left\{\left(1+\frac{U}{\varepsilon}\right)\lg\left(1+\frac{U}{\varepsilon}\right) - \frac{U}{\varepsilon}\lg\frac{U}{\varepsilon}\right\}$$

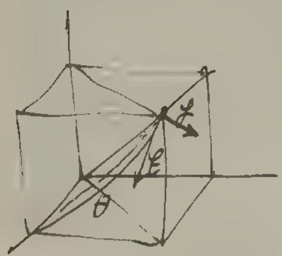
= entropia średnia jednego rezonatora

W razie dużych r:

$$X = \frac{xz}{c^2r^3}\ \ddot{f}\left(t-\frac{r}{c}\right) \qquad L = -\frac{y}{c^2r^2}\ \ddot{f}$$

$$Y = \frac{yz}{\cdot\cdot}\ \cdot\cdot \qquad M = \frac{x}{c^2r^2}\ \ddot{f} \qquad \mathfrak{E}_R = \frac{2R}{c^2r^3} \qquad \mathfrak{H}_R = -\frac{R}{c^2r^2}$$

$$Z = -\frac{x^2+y^2}{c^2r^3}\ \cdot\cdot \qquad N = 0$$



symetrja względem z

więc w płaszczyźnie $y=0$

$$X = \frac{xz}{c^2r^3} \qquad L = 0$$

$$Y = 0 \qquad M = \frac{x}{c^2r^2}$$

$$Z = -\frac{x^2}{c^2r^3} \qquad N = 0$$

$$\sqrt{X^2+Z^2} = \frac{x}{c^2r^2}$$

$r \perp \mathfrak{E} \perp \mathfrak{H}$

$$|\mathfrak{E}| = |\mathfrak{H}| = \frac{\sin\theta}{c^2r}\ \ddot{f}\left(t-\frac{r}{c}\right)$$

Prąd energji Poyntinga $= \frac{c}{4\pi}[\mathfrak{E}\,\mathfrak{H}] = \frac{\sin^2\theta}{4\pi c^3 r^2}\,(\ddot{f})^2$

w całości

$$\frac{2\pi}{4\pi c^3}\int_0^\pi \sin^3\theta\, d\theta\ (\ddot{f})^2 = \frac{1}{2c^3}\int_0^\pi \sin^3\theta\, d\theta$$

$$\sin\theta\, d\theta - \cos^2\theta \sin\theta\, d\theta \qquad \cos\theta - \frac{\cos^3\theta}{3} = \qquad = \frac{4}{3}$$

~~podczas~~ od chwili t do $t+T$:

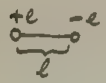
$$= \frac{2}{3c^3}\int_t^{t+T} \ddot{f}^2\left(t-\frac{r}{c}\right) dt$$

więc tym samym czasie ~~[illegible]~~ przez kulę O:

$$\int_{t-\frac{r}{c}}^{t-\frac{r}{c}+T} \ddot{f}^2\left(t-\frac{r}{c}\right) = \int_t^{t+T} \ddot{f}^2(t)\, dt$$

$$= \ddot{f}\dot{f} - \int \dddot{f}\dot{f}\, dt$$

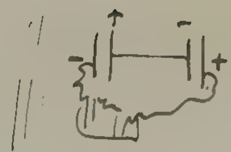
Liniowy oscylator



$el = f$ = moment elektryczny

siła = $X = -f\frac{x}{r^3}$

$Y = \dots$



e^2 energia elektrost.

$i^2 = \left(\frac{de}{dt}\right)^2$ magnet.

$\left.\begin{matrix}+e\\-e\end{matrix}\right\}$ jakoś połączone za pomocą jakiegoś mechanizmu

$U = \frac{K}{2} f^2 + \frac{L}{2} \dot{f}^2$ tak jak w układzie albo dynamice mechanicznej

W razie jeśli energia niezmienna:

$$dU = Kf\,df + L\dot{f}\,d\dot{f} = 0$$

$$Kf + L\ddot{f} = 0$$

$$f = C\cos(2\pi\nu_0 t - \vartheta); \quad \nu_0 = \frac{1}{2\pi}\sqrt{\frac{K}{L}}$$

Rozchodzenie się fali (elektr.) ze źródła

$$F = \frac{1}{r} f\left(t - \frac{r}{c}\right)$$

oś wibratora = Z

$$X = \frac{\partial^2 F}{\partial x \partial z} \qquad L = \frac{1}{c}\frac{\partial^2 F}{\partial y \partial t}$$

$$Y = \frac{\partial^2 F}{\partial y \partial z} \qquad M = -\frac{1}{c}\frac{\partial^2 F}{\partial x \partial t}$$

$$Z = \frac{\partial^2 F}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 F}{\partial t^2} \qquad N = 0$$

$$\frac{\partial X}{\partial t} = c\left(\frac{\partial N}{\partial y} - \frac{\partial M}{\partial z}\right) \qquad \frac{\partial L}{\partial t} = -c\left(\frac{\partial Z}{\partial y} - \frac{\partial Y}{\partial z}\right)$$

$$\frac{\partial Y}{\partial t} = \qquad \vdots$$

$\cancel{\int dt}$ wtedy jeśli $\ddot{f}\frac{\dot{f}}{c^3} \ll U = L\dot{f}^2$

$\sqrt{K}\, f \sim \sqrt{L}\, \dot{f}$ $\quad \frac{\ddot{f}}{c^3} \ll L\dot{f}$

$\sqrt{L}\, \ddot{f} \sim \sqrt{K}\, \dot{f}$ $\quad \therefore \quad \sqrt{K}\, \dot{f} \ll \sqrt{L^3}\, \dot{f}$

$c^3\sqrt{\frac{L^3}{K}} \gg 1 \qquad c^3\sqrt{\frac{K}{L^3}} = \text{mało} = 6$

$\int \dot{f}\, dM_t = \frac{2}{3c^3}\dot{f}\,\dddot{f}$

$\downarrow$

$Kf + L\ddot{f} - \frac{2}{3c^3}\dddot{f} = 0$

w razie zewnętrznych działań jeszcze energia elektryczna $\quad E_z\, e\frac{dl}{dt} = E_z\dot{f}$

więc w takim razie:

$Kf + L\ddot{f} - \frac{2}{3c^3}\dddot{f} = E_z$ $\qquad$ w $\cancel{\ldots}$ x, j

$f = Ae^{\alpha t} \ldots\ldots = Ce^{\alpha t}\cos(\beta t - \vartheta) \qquad \nu_0 = \frac{\beta}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K}{L}}$

$\alpha = -\frac{K}{3c^3L^2}$

$E_z = C\cos\left[2\pi\nu\left(t - \frac{x}{c}\right) - \vartheta\right]$

$\bar{U} = \frac{c^2}{\nu_0^2}\bar{K}_0 \qquad K_0 = 6_0\,\frac{c}{8\pi}$

$\uparrow$ dla drgań.

$$S = \cancel{\lg}\, W$$

$$S_1 + S_2 = f(W_1) + f(W_2) = f(W_1 W_2)$$

$$S = k \lg W$$

$$f(x\, y\, z\; \xi\, \eta\, \zeta)\; \underbrace{dx\, dy\, dz\, d\xi\, d\eta\, d\zeta}_{d\sigma}$$

$$W = \frac{N!}{\Pi(f\, d\sigma!)} \qquad \cancel{\frac{N!}{1!\,2!\,\ldots}}$$

$$S = k \lg N! - k \sum \lg(f\, d\sigma)! \qquad n! = \left(\frac{n}{e}\right)^n \sqrt{2\pi n} \qquad N = \sum f\, d\sigma$$

$$\lg n! = n(\lg n - 1)$$

$$= k \lg N! - k \sum f\, d\sigma \left[\lg(f\, d\sigma) - 1\right] \quad = \text{const} - k \sum f\, d\sigma \lg(f\, d\sigma) =$$

$$= \text{const} - k \int f \lg f\, d\sigma$$

$$\delta S = 0 \qquad \int (\lg f + 1)\, \delta f\, d\sigma = 0$$

$$\int \delta f\, d\sigma = 0$$

$$\int (\xi^2 + \eta^2 + \zeta^2)\, \delta f\, d\sigma = 0 \quad \Bigg| \quad \lg f + \beta(\xi^2 + \eta^2 + \zeta^2) = \text{const}$$

$$f = \alpha e^{-\beta(\xi^2 + \eta^2 + \zeta^2)}$$

$$V = \int dx\, dy\, dz$$

$$N = \int f\, d\sigma = V\alpha \int e^{-\beta(\xi^2 + \eta^2 + \zeta^2)}\, d\xi\, d\eta\, d\zeta = V\alpha \left(\frac{\pi}{\beta}\right)^{3/2}$$

$$U = \frac{m}{2} \int (\xi^2 + \eta^2 + \zeta^2)\, f\, d\sigma = \frac{Vm\alpha}{2} \int (\xi^2 + \eta^2 + \ldots)\, e^{\ldots}\, d\xi \ldots$$

$$= \frac{3}{4}\, V m \frac{\alpha}{\beta} \left(\frac{\pi}{\beta}\right)^{3/2}$$

$$\alpha = \frac{N}{V} \left(\frac{3mN}{4\pi U}\right)^{3/2} \qquad \beta = \frac{3mN}{4U}$$

$$S = \text{const} + kN\left(\frac{3}{2} \lg U + \lg V\right)$$

Gas jednoatomowy

4 atomów 3 komory

abcd I II III

możliwe wypadki:

004	013	022	112
040	031	202	121
400	103	220	211
	130		
	301		
	310		

Notation: distribution 004 013 022 112

$\begin{Bmatrix} 0e & 1e & 2e & 3e & 4e \\ 2 & 0 & 0 & 0 & 1 \end{Bmatrix}$

1 1 1 0 1 0

te kategorie są między sobą ~~...~~ jednakowo prawdopodobne w każdej klasie, ale nie zapominać

bo np. 004 może być zrealizowane tylko w jeden sposób

a	III
b	III
c	III
d	III

$W_0 = \left(\frac{1}{3}\right)^4$

podczas gdy np. 022 w następujące sposoby

a	II	II	II	III	III	III
b	II	III	III	II	II	III
c	III	II	III	II	III	II
d	III	III	II	III	II	II

Wsp. prawdop. klas:

$\left(\frac{1}{3}\right)^4 \quad 4\left(\frac{1}{3}\right)^4 \quad 6\cdot\left(\frac{1}{3}\right)^4 \quad 12\left(\frac{1}{3}\right)^4$

w całości totalnie

$$\frac{3 + 4\cdot 6 + 3\cdot 6 + 3\cdot 12}{3^4} = \frac{\begin{array}{r}45\\36\\\hline 81\end{array}}{3^4} = 1$$

$W = 6\cdot\left(\frac{1}{3}\right)^4 = \cancel{\ldots}$

ogólnie $\frac{W}{W_0} = \frac{4!}{\alpha!\beta!\gamma!}$ $\Big|$ $\frac{4!}{0!\,2!\,2!} = 6$

$\frac{4!}{1.1.3!} = 4$

$\frac{4!}{1.1.2!} = 12$

rozróżniamy komory, ale ~~...~~ indywidualność cząstki jest obojętna

chodzi o ~~prawdop.~~ rozróżnianie przy danej liczbie atom. i danej liczbie komór, ale dla różnych układów

Planck jedzie w porównywaniu liczy liczbę tych wszystkich kompleksyj

tj. kombinacyj <u>z powtórzeniem</u> (bez względu na porządek)

I I I I	I II II II	II II II II	III III III III
I I I II	I II II III	II II II III	
I I I III	I II III III	II II III III	
I I II II	I III III III	II III III III	
I I II III			
I I III III			

$\Big\} = 15$

ogólnie $\dfrac{(N+P-1)!}{P!\,(N-1)!}$

~~a~~ uważając je za równie prawdopodobne

a to może za miarę prawdopodob. że N elementów rozdziela się na P rezonatorów

więc nie liczby N P są dane tylko

liczba P jest dana N jest zmienne!

i porównuje się prawdop. dla różnych N

On jedzie tak w takim razie uważa co do tego, jakie wszystkie są równie

prawdopodobne.

Chyba tak:

Dana liczba P rezonatorów, indywidualnie oznaczonych

Dla każdego z nich przypisujemy liczbę ~~elementów~~ energii wedle przypadku, ciągnąc z urny

liczby od $0 - M$

potem porządkujemy wszystkie wedle <u>sumy</u> energii otrzymanej

Najmniejsza energia 0, największa MP

Równie prawdop. będzie również np.

004 jak 005 albo 003

ale tylko gdy suma jest ~~5~~ 5 jest więcej niż tylko gdy suma = 4

Różni się o stosunku więcej liczb kompleksy

$$\frac{\frac{7!}{5!\,2!}}{\frac{6!}{4!\,2!}} = \frac{7}{5} \qquad \frac{\frac{6!}{2!\,4!}}{\frac{5!}{2!\,3!}} = \frac{6}{4} = \frac{3}{2}$$

003
030
300
102
120
111
201
210
012
021

Ale

Jeśli jak u Plancka N, P dane to
nie ma wcale sensu mówić o prawdopodob.

Tak chodzi zatem o prawdop. osiągnięcia pewnej całkowitej ilości energii, jeśli każdy oscylator wypełnia przypadkowo więcej albo mniej niż połowę

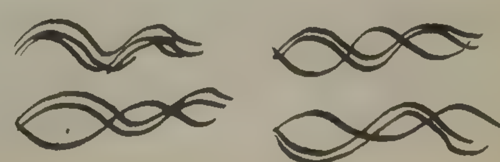
[Czy to w pewnym obrazie nie będzie wyrażone Różni interferencyjnego wektorów przypadkowego?]

Jedno źródło E = 1

Dwa źródła

E = 0
albo E = 4

Jeśli trzy źródła, to E = 9
1
1
1



$$\lg f = -\frac{N}{R\theta}(L+U) = \frac{1}{k}S$$

$J = \frac{C\cancel{\lambda}}{\lambda^5 (e^{\frac{c}{\lambda\theta}}-1)}$ $\frac{\partial J}{\partial \lambda} = 0$: $5 = \frac{c}{\lambda T}\,\frac{1}{1-e^{-\frac{c}{\lambda T}}}$ $c = 4.965\lambda_m T$

$c = 14598$

Rubens – Kurlbaum

Fluorit 24.0 μ 31.6 μ   NaCl 51.2 μ

−273°

$$e = \frac{C}{\lambda^5}\left[e^{\frac{c}{\lambda\theta}} - 1\right]^{-1} = \frac{C}{\lambda^5} e^{-\frac{c}{\lambda\theta}}\left[1 - e^{-\frac{c}{\lambda\theta}}\right]^{-1} \neq \frac{C}{\lambda^5} e^{-\frac{c}{\lambda\theta}} \quad \text{Wien} \quad \frac{\lambda\theta}{c} \gg 1$$

$$e = \frac{C}{\lambda^5}\left[1 + \frac{c}{\lambda\theta} - 1\right]^{-1} = \frac{C\theta}{c\lambda^4} \quad \text{Rayleigh} \quad \frac{\lambda\theta}{c} \ll 1$$

$\frac{c}{\lambda\theta} = x$ $\lambda = \frac{c}{\theta x}$ $d\lambda = -\frac{c\,dx}{\theta x^2}$

$$\int J\,d\lambda = \frac{C\theta^4}{c^4}\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{C\theta^4}{c^4}\int_0^\infty x^3\,dx\left(e^{-x} + e^{-2x} + e^{-3x} + \dots\right]$$

$$\int_0^\infty x^3 e^{-nx}\,dx \qquad = \frac{C\theta^4}{c^4}\cdot 6\left[1 + \left(\tfrac{1}{2}\right)^4 + \left(\tfrac{1}{3}\right)^4 + \dots\right]$$

$$\frac{1}{n^4}\int (nx)^3 e^{-nx}\,d(nx) =$$

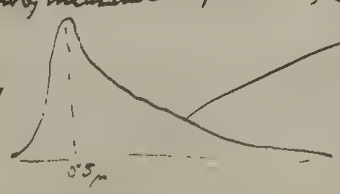
$$= 6.4938\cdot\frac{C\theta^4}{c}$$

$$\frac{1}{n^4}\int x^3 e^{-x}\,dx = \frac{3!}{n^4}$$

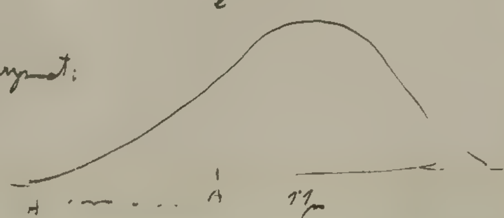
6490°C $\theta = 5607$°C

3 [illegible] temperatur 1) θ^4 2) $\lambda_m\theta$ 3) $\sim\frac{1}{e^{\frac{c}{\lambda\theta}}-1}$ 4) $J_m \sim \theta^5$

Langley



[illegible] pigment:



$$f(u,v,w) = A e^{-\frac{N}{H\theta}\left[\frac{m}{2}(u^2+v^2+w^2) + \Phi\right]}$$

N.p. [illegible]

$$\Phi = mgz$$

$N = 8 \cdot 3 . 10^7$

$H = 7 . 10^{23}$

$$\frac{Nm}{H\theta} gz = \frac{gz}{R\theta}$$

$$E_\lambda^{d\lambda} = \frac{c^2 h}{\lambda^5} \frac{d\lambda}{e^{\frac{ch}{k\lambda\theta}} - 1}$$

$$\frac{c^2 h}{\lambda^5} e^{-\frac{ch}{k\lambda\theta}}$$

$$\frac{c k \theta}{\lambda^4}$$ Rayleigh

$k = 1{\cdot}35 \; 10^{-16} \left(\frac{\mathrm{erg}}{\theta}\right) = \frac{2}{3}\left(m\frac{v^2}{2}\right)_{1^\circ} = \frac{H}{N}$

$h = 6{\cdot}55 . 10^{-27}$ erg sec

$e = 9658 . 3 . 10^{10} . \frac{k}{R} = 4{\cdot}69 . 10^{-10}$

$\lambda = \frac{c}{\nu}$

[illegible]

$$E_\lambda^{d\nu} = \frac{h}{c^2} \frac{\nu^3 d\nu}{e^{\frac{N}{H\theta}(\nu h)} - 1}$$

Vibr. energy $\quad 0, \; \varepsilon, \; 2\varepsilon \dots$

pravdp. $\quad 1 : e^{-\frac{\varepsilon}{k\theta}} : e^{-\frac{2\varepsilon}{k\theta}} \dots$

$$1 + x + x^2 + x^n \dots = \frac{1}{1-x}$$

$$1 + 2x + 3x^2 + \dots = \frac{1}{(1-x)^2}$$

N [illegible] : $N = M\left[1 + e^{-\frac{\varepsilon}{k\theta}} + e^{-\frac{2\varepsilon}{k\theta}} + \dots\right] = \frac{M}{1 - e^{-\frac{\varepsilon}{k\theta}}}$

energie celkova : $E = M\varepsilon\left[e^{-\frac{\varepsilon}{k\theta}} + 2e^{-\frac{2\varepsilon}{k\theta}} + \dots\right] = M\varepsilon \frac{e^{-\frac{\varepsilon}{k\theta}}}{(1-e^{-\frac{\varepsilon}{k\theta}})^2}$

$$\frac{E}{N} = \varepsilon \frac{e^{-\frac{\varepsilon}{k\theta}}}{1 - e^{-\frac{\varepsilon}{k\theta}}} = \frac{\varepsilon}{e^{\frac{\varepsilon}{k\theta}} - 1}$$

$$6 = \frac{\nu^2}{c^2} E = \frac{\nu^3}{\dots} d\nu$$

$\nu = \frac{c}{\lambda}$

$d\nu = \frac{c}{\lambda^2} d\lambda \qquad \lambda^5 \; d\lambda$

$$E = \frac{N\varepsilon}{e^{\frac{\varepsilon}{k\theta}} - 1} = \frac{Nh\nu}{e^{\frac{h\nu}{k\theta}} - 1}$$

$h = 6{\cdot}55 \cdot 10^{-27}$

$$\frac{6{\cdot}5 \cdot 3 \cdot 10^{10} \cdot 10^{-27}}{59 \cdot 10^{-5}} = 3 \cdot 10^{-12}$$

$h\nu = 6{\cdot}55 \cdot 10^{-27} \cdot \frac{c}{\lambda} = \frac{10^{-4} \cdot 0{\cdot}59}{3 \cdot 10^{10}} = 3 \cdot 10^{-12}$ erg.

Sposoby obserwacji

1). Oko ludzkie czerw. 0·82 – 0·62

żółt – 0·56

ziel – 0·50

nieb – 0·45

fiolet. – 0·38

2). Fluorescencja

3). Fotografia, w ogóle fotochemiczne wpływy; aż do 0·1

Schumann Kwarcowe soczewki, wodór, ewakuacja, płyty bez żelatyny

Ale nie tylko poza fiolet.; sensybilizatory (Abney) aż do 2·7

Eozyny, Cyanina, etc.

4). Cieplne (γ) Radyometr δ. Radiomikrometr Boys

Rubens & Nichols

Thermometr

α). Stos termoelektr. Melloni Rubens Linear Thermosäule 20 Fe-Konstantan (10^{-6}° = 1 mm) w żadnej zaczynie Lebedew ewak.

β). Bolometr Langley 1881

Lummer & Kurlbaum

60Ω (Pt 1μ)

czułość aż do 10^{-6}°

Paschen $\Delta\theta = 10^{-7}$°

Aż do 60μ Rubens Nichols (1897) Reststrahlen

Rubens Aschkinass (1898) Palnik Auera (bez szkiełka)

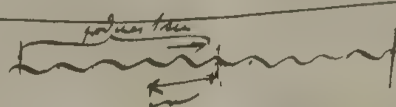
od Lymana 5 rzędu długości $\lambda = 0{\cdot}061\mu$

300μ Rubens & .. (1911) Znamy przynajmniej takie długości, ale inne zjawisko

Najkrótsze elektryczne Lampa, Lebedew 6 mm

Zasada Dopplera (1842)

podłużne fale



$\lambda = c\tau$

$n = \frac{1}{\tau}$

$n' = n\left(1 + \frac{v}{c}\right) = \frac{1}{\tau'}$

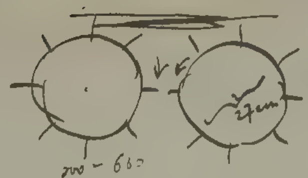
$\frac{\tau'}{\tau} = \frac{1}{1+\frac{v}{c}} \neq 1 - \frac{v}{c}$

$\frac{\lambda'}{\lambda} = 1 + \frac{v}{c}$

Doświadczenie akustyczne Buys Ballot, Vogel lokomot. ~~Fizeau~~ Fizeau – rok później wersja 1·051 1·050 Kongdodziene

Bielopolski

[illegible] [illegible]



fotograf.

30–50 obrotów / sek.

zwierciadła 2 × 10·5 × 0·3 cm

prędkość środka 700 $\frac{m}{sec.}$

$$\frac{\frac{700.12}{8.400}}{3.10^8} = \mp 3.10^{-5} \Big\} = \text{stosunkowe przesunięcie } \frac{d\lambda}{\lambda}$$

podwójny Na:D $\begin{array}{l} 0{\cdot}5896\,156 \\ \underline{0{\cdot}5890\,188} \end{array}$ 10^{-3} więc $\frac{1}{30}$ część odstępu linii podwójnej!

~~[illegible]~~

to byłoby jeszcze względnie dostrzegalne w lepszych spektroskopach, ale ta trudność że względ zaniechany wskutek ostrożności

zmian gęstości potwierdzenie jakościowe. Ilościowo ± 20%

Słońce Crew, Dunér $\left\{ \begin{array}{ll} 0^\circ & \text{---} \ 25{\cdot}46^d \\ 45^\circ & 30{\cdot}03 \\ 75^\circ & \cancel{33{\cdot}4}4 \end{array} \right.$

Kometa 1882 Thollon & Gouy 61–76 km spektr.
73 km astron.

Saturn Keeler (1895) obrazy z pierścieni [illegible] [illegible]

♀

Algol (1889) Vogel podwójny

Inne gwiazdy podwójne spektroskop.

Stark 1905 Kanalstrahlen H_2 N_2 Hg Na K $5-6.10^7$ $\frac{e}{m}$ v [illegible]

Przy ozsunięciu tłoka zmienia się barwa i temperatura promieniowania [illegible]

$$\frac{\delta\lambda}{\lambda} = \cancel{2\frac{u}{c}} \cancel{\frac{2\,dx}{\tau c}}$$

$$\frac{d\lambda}{\lambda} = 2\frac{u}{c}\cdot \nu^2 \quad \text{liczba odbić}$$

$$\nu = \frac{\frac{dx}{u}}{\frac{2x}{c}} = \frac{dx\,c}{2x\,u}$$

$$\frac{d\lambda}{\lambda} = \frac{dx}{x}$$

ale to tylko trzecia część promieni się przesuwa $d\lambda$:

$$\frac{d\lambda}{\lambda} = \frac{1}{3}\,\frac{dx}{x}$$

Temperatura:

Gęstość promieniowania (Energia po $1\,cm^3$): $\psi(\theta)$ $\qquad \psi = c\,\theta^4$

$$E = q\,x\,\psi$$

po osunięciu tłoka : praca wykonana podczas sunięcia $\qquad dE = \pm q\,\frac{\psi}{3}(-dx)$

$$E' = q(x+dx)(\psi+d\psi)$$

$$E'-E = q\,x\,d\psi + q\,\psi\,dx = -q\,\frac{\psi}{3}\,dx$$

$$\frac{d\psi}{\psi} = -\frac{4}{3}\,\frac{dx}{x} = -4\,\frac{d\theta}{\theta}$$

$$\frac{1}{3}\,\frac{dx}{x} = -\frac{d\theta}{\theta}$$

zatem $\frac{d\lambda}{\lambda} = -\frac{d\theta}{\theta}$

$$\lambda = \frac{\alpha}{\theta}$$

$$\frac{d\lambda}{d\theta} = -\frac{\alpha}{\theta^2}$$

$$\lambda\,\theta = \text{const.}$$

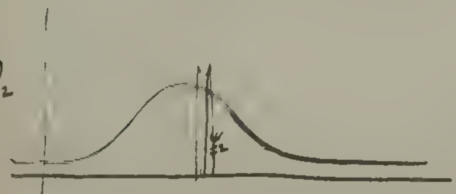
Wien'sches Verschiebungsgesetz

Znając promieniowanie dla jednej temp. (ciała czarnego) możemy obliczyć dla jakiejś bądź innej:

θ_1

po zgęszczeniu:

θ_2



$$\lambda_2 : \lambda_1 = \theta_1 : \theta_2$$

$$d\lambda_2 : d\lambda_1 = \theta_1 : \theta_2$$

$$\int \psi_2 \, d\lambda : \int \psi_1 \, d\lambda = \theta_1^4 : \theta_2^4$$

zatem $\psi_2 : \psi_1 = \theta_2^5 : \theta_1^5$

Wien: $\psi(\lambda, \theta) = \dfrac{c_1 e^{-\frac{c_2}{\lambda \theta}}}{\lambda^5}$

$\lg \psi = \ldots$

$\dfrac{\partial \psi}{\partial \lambda} = 0$ $\qquad \lambda_{max}\,\vartheta = \text{const} = 29[illegible]\ (\mu)$

słońce [illegible] (Langley)

$\lambda_m = 0{\cdot}5\,\mu$

$\theta = 5774^\circ = 5501^\circ C$

Langley [illegible] $-2[illegible]^\circ$

względem -20°

$= [illegible]\ 12{\cdot}2\,\mu$

podczas gdy rachunek $10{\cdot}7\,\mu$

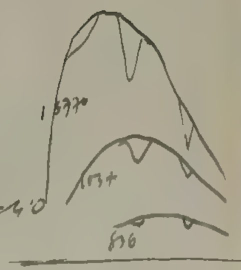
dla [illegible] [illegible]

Planck

Paschen $J = C \dfrac{e^{-\frac{c}{\lambda\theta}}}{\lambda^{\alpha}}$

α vyšel 5·09

sodík, CuO 5·62



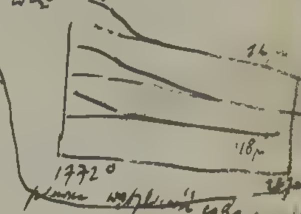
Lummer, Pringsheim 1897 → α = 3·96

1μ – 6μ

jistě po opravách α = 4·0

Paschen 1897 ale nevyhovuje

Lummer – Pringsheim 1900 až do 18μ (fluoritparanol) ϑ = −180° to 1383° || $\log J = \alpha - \frac{\beta}{\theta}$ (Isochromaty)



Planck 1900 $J = \dfrac{C\lambda^{-5}}{e^{\frac{c}{\lambda\theta}} - 1} = \dfrac{C}{\lambda^5\left[e^{\frac{c}{\lambda\theta}} - 1\right]}$

Rubens & Kurlbaum 1900 − 188° až do 1500°

Paschen 1901 — Reststrahlen ... 8·85μ

Flussspat 24, 31·6

Reststrahlen von Steinsalz λ = 51·2μ

ϑ	J	Wien	Planck
−273		−121·5	−23·8
−188	−20·6	−107·5	−21·9
−80	−11·8	−48·0	−12·0
+20	0	0	0
250	34·0	63·5	30·4
500	64·5	90	63·8
750	98·7	118	93·2
1000°	132·0	132	132
1500	196·8	147·5	200
1250°	164·5	141	166

$\lambda T \gg 1$

$J = \dfrac{c\,T\,e^{-\frac{c}{\lambda T}}}{\lambda^4}$

$C = 3{\cdot}7179 \cdot 10^{-5}$ (erg cm²/sec)

$c = 1{\cdot}4598$ (cm grad)

$\lambda_m \theta = 2940\ \mu°$

525 poč. žár. ~~[illegible]~~

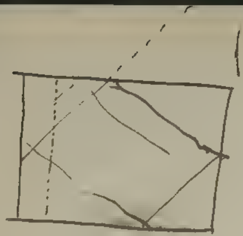
700 tmavočerv.

1000 jasné

1200 jasné pomeranč.

1300 bílo-žlutá

1500 jiskrné



Jeśli promień nachylony pod kątem θ to skrócenie

$$\frac{d\lambda}{\lambda} = 2\frac{u}{c}\cos\theta$$ przy jednem odbiciu

~~podczas tak [illegible]~~.

ale podczas drogi dx będzie $\frac{c\,\Delta t}{x\cos\theta}$ odbić

więc $$\frac{d\lambda}{\lambda} = \frac{2u}{c}\cos\theta \cdot \frac{c\cos\theta}{x}\,\frac{dx}{u} = \frac{dx}{x}\cos^2\theta$$

przeciętnie zatem;
$$\overline{\left(\frac{d\lambda}{\lambda}\right)} = +\frac{dx}{x}\,\frac{\int_0^{\frac{\pi}{2}} 2\pi\sin\theta\,d\theta\cos^2\theta}{\int_0^{\frac{\pi}{2}} 2\pi\sin\theta\,d\theta} = \frac{1}{3}\frac{dx}{x}$$

ale de facto nie rozdzielą się promienie na 3 kategorje, tylko jeśli ich dostatecznie rozłożony to równomiernie (jeśli ścianki nie są ∞ gładkie i t.d.)

$$\frac{d\lambda}{\lambda} = +\frac{1}{3}\frac{\delta x}{x} \qquad \frac{d\theta}{\theta} =$$

~~Ilość energii zmniejsza się i t.d.~~

~~$\varphi = (\varphi - d\varphi)\,dx$~~

~~$\psi = \frac{\psi}{3}dx + \frac{d\theta}{\theta}\ \ldots$~~

$$\frac{d(\varphi x)}{\cancel{dx}} = -\frac{\varphi}{3}dx$$

$$\varphi\,dx + x\,d\varphi = -\frac{\varphi}{3}dx$$

$$\frac{d\varphi}{\varphi} = -\frac{4}{3}\frac{dx}{x} = 4\frac{d\theta}{\theta} = -4\frac{d\lambda}{\lambda}$$

$\lambda\theta = \text{const}$

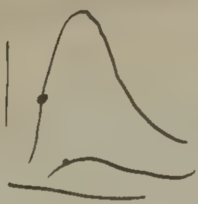
$$f(\lambda,\theta) = \varphi\left(\frac{\lambda}{\theta}\right)$$

$\frac{\psi}{\theta^4}$ to samo dla θ

$$\frac{\psi(\theta)}{\theta^4} = \int_0^\infty \frac{\varphi(\lambda\theta)}{\theta^5}\,d(\lambda\theta)$$

$f(\lambda\theta)$

2887



$\psi_{\theta_0} = f_c(\lambda)$

$\psi = \cancel{f(\theta,\lambda\theta)}\ f(\theta)\cdot\varphi(\lambda\theta)$

$\cancel{\int} \cancel{\int f(\theta,\lambda\theta)\,d\lambda} =$

$$\int_0^\infty f(\theta)\,\varphi(\lambda\theta)\,d\lambda = C\theta^4$$

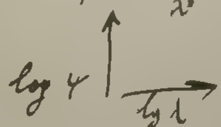
$$= \frac{f(\theta)}{\theta}\int_0^\infty \varphi(x)\,dx$$

$$\therefore f(\theta) \sim \theta^5$$

$$\therefore \psi = \theta^5\varphi(\lambda\theta)$$

Albo takie $\psi = \dfrac{\varphi(\lambda\theta)}{\lambda^5}$

Paschen



blachy pokryte sadzą Fe_2O_3 CuO

Pt wzięli

$$J = c\,\frac{e^{-\frac{c_2}{\lambda\theta}}}{\lambda^\alpha} \quad \Big/ \alpha = 5{\cdot}3 - 6{\cdot}4 \quad Pt$$

Wien 1896 $\alpha = 5$

Planck 1899

[illegible]

$$\log\psi = \log\varphi(\lambda\theta) - 5\lambda$$

$$= \Phi(\lambda\theta) - 5\lambda$$

$$\log\psi + 5\lambda = \Phi(\lambda\theta)$$

$$= \Phi\,\log(\lambda\theta)$$

$$= \Phi(\log\lambda + \log\theta)$$

[illegible] w.f. λ, θ [illegible]

[illegible] co można [illegible]

Gęstość promieniowania niezależna od źródła

Ponieważ gęstość [illegible] i pewien skład ; gęstość charakteryzuje temperaturę prom. jeśli skład określony od ⌒ to można mówić o mieszaninie promieniowania o różnej temp.

Niemożliwość ogrzania jakimkolwiek sposobem do wyższej temp. niż źródło promienia.

Ciśnienie promieniowania

Bartoli 1876 : Gdyby nie ogrzanie ciśnienia to można by za pomocą tłoku i cylindra promieniowanie przenosić z ciała chłod. do ciepł. bez kompensaty

Maxwell 1873

$$Z = A \sin \alpha\left(t + \frac{x}{v}\right) \qquad M = A\sqrt{\frac{K}{\mu}} \sin \alpha\left(t + \frac{x}{v}\right)$$

$$X = Y = 0 \qquad L = N = 0$$

Energia $W = \frac{1}{8\pi}\left(K Z^2 + \mu M^2\right)$ $\boxed{\text{na Vol. jedn.}}$

$= 2\frac{K}{8\pi} A^2 \sin^2 \alpha\left(t + \frac{x}{v}\right)$

$P = W$

$\overline{P} = \overline{W} = \frac{K}{8\pi} A^2$

$$K \frac{\partial Z}{\partial t} = c\left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}\right) \qquad \frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} = 0$$

$$\mu \frac{\partial M}{\partial t} = -c\left(\frac{\partial X}{\partial z} - \frac{\partial Z}{\partial x}\right) \qquad \frac{\partial L}{\partial x} + \frac{\partial M}{\partial y} + \frac{\partial N}{\partial z} = 0$$

$$\mu K \frac{\partial^2 Z}{\partial t^2} = + c^2 \frac{\partial^2 Z}{\partial x^2} \qquad \mu \alpha = \frac{\alpha}{v}$$

$$v = \frac{c}{\sqrt{\mu K}}$$

~~(Poynting)~~

$$\cancel{= \frac{1}{4\pi}(Z M)}$$

$$\cancel{\overline{W} = \frac{A^2}{4\pi}\sqrt{\frac{K}{\mu}} \sin \alpha\left(t + \frac{x}{v}\right)}$$

$$\cancel{\overline{W} = \frac{A^2}{8\pi}\sqrt{\frac{K}{\mu}}}$$

Przypływ energii pro sek. $= v\overline{W} = \frac{2 \text{ gcal.}}{60}$ słońce

$$\overline{W} = \frac{2 \cdot 4{,}2 \cdot 10^7}{60 \cdot 3 \cdot 10^{10}} = \frac{8{,}4 \cdot 10^{-5}}{1{,}8}$$

$2\overline{P}$ pro m² = 1 dyn

$x = 0 \qquad Z = 0$

$$Z = A\left[\sin \alpha\left(t + \frac{x}{v}\right) + \sin \alpha\left(t - \frac{x}{v}\right)\right] = 2A \sin \alpha t \cos \alpha \frac{x}{v}$$

$$M = A\sqrt{\frac{K}{\mu}}\left[\sin \alpha\left(t + \frac{x}{v}\right) - \sin \alpha\left(t - \frac{x}{v}\right)\right] = 2A\sqrt{\frac{K}{\mu}} \cos \alpha t \sin \alpha \frac{x}{v}$$

$$W = \frac{K}{8\pi} Z^2 + \frac{\mu}{8\pi} M^2 = 4\frac{A^2 K}{8\pi}\left[\sin^2 \alpha t \cos^2 \alpha \frac{x}{v} + \cos^2 \alpha t \sin^2 \cdots\right]$$

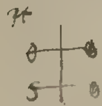
$$\overline{W} = A^2 \frac{K}{4\pi}$$

Ciśn. prostopadł. padanie bez sił

absorb. P

zwier. $2P$

Lebiediew 1900 · Nichols i Hull 1903 Ann. 12



jedna blaszka 0'1 mm

druga " 0'02 " ← siły radiometr. 5 razy tak duże ewakuujemy ciśnienie

Poynting i Barlow siły styczne $\xrightarrow[\text{powierzchnia}]{}$ (Poynting The pressure of light 1910)

momenty obrotowe

reakcja źródła światła

Lebiediew ciśnienie gaz absorbujący ; Komety! Arrhenius Schwarzschild etc.!

Jeśli promieniowanie w wszystkich kierunkach

$$P = 2\pi \frac{\sin\varphi\, d\varphi\; dS\cos\varphi}{c}\cdot \overset{e}{\left(\frac{\eta}{4\pi\alpha}\right)}\, 2\cos\varphi \;\Big|\; = \frac{4\pi e}{c}\,\frac{1}{3}$$

$$= \frac{dS.\,\eta}{\alpha c}\cdot\frac{1}{3} = \frac{u}{3}$$

Gęstość

Vyzáření energie při oscilatorním proudu.

$$\eta \, \frac{\cancel{}}{4\pi r^2} \, e^{-\alpha r} \, \frac{\alpha^2 \, d\omega \, dr}{c}$$

$$6 = \frac{\eta}{4\pi\alpha c} \int_0^{\pi} \underbrace{2\pi \sin\theta \, d\theta} = \frac{\eta}{\alpha c} = \cancel{}$$

$$e_{pov.} = \frac{\eta}{4\pi\alpha}$$

Účinení promien.

$$P = \frac{\eta}{4\pi\alpha c} \, 2\int_0^{\frac{\pi}{2}} 2\pi \sin\theta \cos^2\theta \, d\theta = \frac{\eta}{\alpha c} \cdot \frac{1}{3} = \cancel{} \frac{1}{3} = \frac{6}{3}$$

$x \, d6_1$

$$d((x+\delta x)6) + \frac{6_1}{3} d\delta x = 0$$

$$d(x 6_1 + \frac{6}{3} \ldots$$

$$\cancel{U = v\, 6 = \text{konst}\, v_0 6_0}$$

$$\cancel{\delta\varphi = v\, d6 + P\, dv = v\, d6 + \frac{6}{3} dv}$$

$$\cancel{\frac{\delta\varphi}{\theta} = dS}$$

$$\cancel{\frac{v}{\theta} \frac{\partial 6}{\partial \theta} = \theta \frac{\partial S}{\partial \theta}}$$

$$\cancel{\frac{v}{\theta} \frac{\partial 6}{\partial v} + \frac{6}{3} = \theta \frac{\partial S}{\partial v}}$$

$$\cancel{v \frac{\partial^2 6}{\partial \theta \partial v} + \frac{1}{3} \frac{\partial 6}{\partial \theta} - \frac{\partial 6}{\partial \theta} = \frac{\partial S}{\partial v}}$$

$$\theta_1: \quad x\, 6_1 + \frac{6_1}{3} x$$

$$\frac{6_1}{3} dx = - x \, d6_1$$

$$dW = \frac{6_1}{3}(x + dx) - \frac{(6_1 - d6_1)}{3} = \frac{d6_1}{3} x$$

$$dW = \frac{4}{3} 6_1 x \cdot \frac{d\theta}{\theta} = \frac{1}{3} x_1 \, d6_1$$

φ

$$4 \frac{d\theta}{\theta} = \frac{d6}{6}$$

$$\lg 6 = \lg \theta^4$$

$$6 \sim \theta^4$$

$$a f(\theta) + a \frac{f(\theta)}{3}$$

$$d[(a+x) f(\theta)] = -\frac{f}{3} dx \quad \frac{1}{3} f(dx) + \alpha \, df = 0$$

$$\cancel{a f(\theta) + a f(\theta)} \quad a + d$$

$$(a + x) \cdot f$$

[illegible]

$\psi = \kappa T^4$ Stefan (1879)

1) Že [illegible] [illegible] [illegible] [illegible] [illegible] 2 [illegible]

[illegible] Bartoli, Boltzmann

[illegible]

$p = E$

$p = \frac{2 \cdot 42 \cdot 10^7}{3 \cdot 10^{10} \cdot 60} = \frac{42}{9} \cdot 10^{-4} = 5 \cdot 10^{-5}$

$\frac{1}{30} \frac{\text{gcal}}{\text{sec}}$

$$A = \frac{m\,l}{l}$$

0.5896156

1980

Rowland

Pryzmat

Pozycja ~~...~~ odchyl[enia] minimalnego

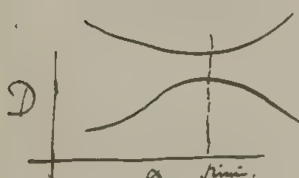
1 / 1

bo wtedy równe D dla nieco różnych —

zatem to mi szybsze że pulary promieni nieco różnymi

mniej zależy ściśle od ustawienia pryzmatu

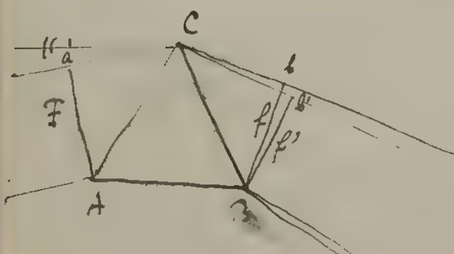
że pozycja my ścisły musi być max albo minim. to wynika z odwracalności pro



Zdolność rozszczep.

dwa rodzaje światła λ λ'

czas potrzebny jednakowy



$$n\,AB = aC + Cb$$

$$n'\,AB = aC + Cb'$$

$$(n'-n)\,AB = C(b'-b)$$

$$\alpha = \frac{Cb' - Cb}{Bb} = \frac{(n'-n)\,AB}{Bb}$$

ćwiczymy prążek pierwszy:

$$\beta = \frac{\lambda}{Bb}$$

osobno tylko będzie widać jeśli $\alpha > \beta$

$$(n'-n)\,Ab > \lambda$$

$$Ab > \frac{\lambda}{n'-n}$$

podstawa ilościowości szeroko, z drugiej strony chwiejna Flint

$n = 1{\cdot}650$

$dn = 0{\cdot}000055$

$\lambda = 0{\cdot}000589$ mm

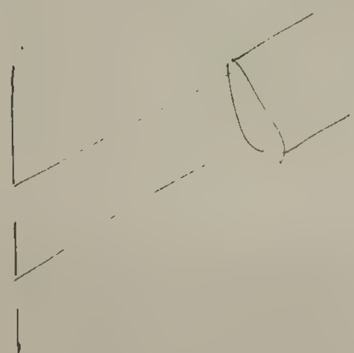
AB przynajmniej 1 cm

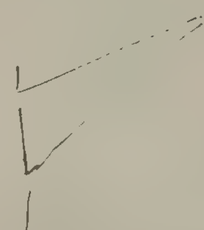
widmo zależy od dyspersji substancji

Pryzmaty [illegible] widma [illegible] — współcz. załam. $n_D - 1$

à Vision directe

	C	D	F	H	$\nu = \frac{n_F - n_C}{n_D - 1}$
Crown	1·515	1·518	1·524	~~1·533~~	0·0166
Flint	~~1·570~~	~~1·575~~	~~1·585~~	~~1·599~~	
	1·614	1·620	1·631	~~1·653~~	0·0276





$\beta = \frac{\lambda}{B}$

$\cancel{\lambda} =$

$\delta\beta = \frac{d\lambda}{b}$

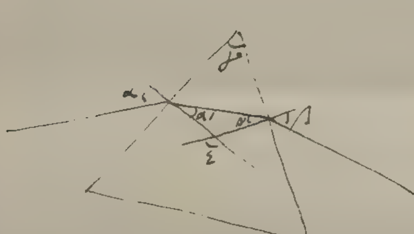
$\delta\gamma = \frac{\lambda}{B}$

$\delta\gamma > \delta\beta$

$\frac{d\lambda}{b} > \frac{\lambda}{B}$

$d\lambda > \frac{b\lambda}{B} = \frac{\lambda}{m}$

Na $\frac{d\lambda}{\lambda} = 0{\cdot}001$ $m = 1000$



$$\left.\begin{aligned} \varepsilon = \alpha' + \beta' &= 180 - \gamma \\ \sin\alpha &= n \sin\alpha' \\ \sin\beta &= n \sin\beta' \end{aligned}\right\}$$

102

Gęstość energii 1 mg 1 cm²

$$\rho = \int \frac{\eta\, dv}{4\pi r^2}\,\frac{e^{-\alpha r}}{c} = \frac{\eta}{c}\int_0^\infty e^{-\alpha r}\,dr = \frac{\eta}{\alpha c}$$

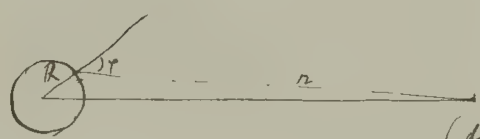
$0 \cdot 0001 .\ r^2 = r^3$

podczas gdy dla zdolności emisyjnej powierzchni na całości $e = \frac{\eta}{4\alpha}$

więc $\rho = \frac{4e}{c}$

$\frac{\varepsilon\, d\omega}{\pi}$

$$\underbrace{i\int_0^{\frac{\pi}{2}} 2\sin\varphi\,\pi\, d\varphi \cos\varphi}_{\Sigma\, i\, d\omega} = i\pi = e$$



$$\int df\, \frac{i\cos\varphi}{4\pi r^2} = \int_0^{\frac{\pi}{2}} \frac{2\pi \sin\varphi\, d\varphi \cos\varphi\; i R^2}{4\pi r^2} = \frac{i R^2}{4 r^2} = S$$

$$e = \frac{4\pi r^2}{R^2} S$$

Ångström 1876

Ångström 188

$\sigma = 1 \cdot 085 \cdot 10^{-12}$

$5 \cdot 32 \cdot 10^{5}$

$4 \cdot 2 \quad 07$

$= 1 \cdot 2 \cdot 10^{12}$

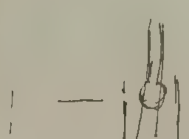
(1880)

Schleiermacher Rosetti

Schneebeli

871

Lummer – Pringsheim 1897



Paschen 1893

$$E_{100} - E_0 = 0 \cdot 0176 \frac{cal}{m}$$

$$e^{-\frac{(x-x_0-\beta x_0\tau-\gamma\tau)^2}{4\tau D}}$$

$$\frac{1}{(2\sqrt{\pi\tau D})}\int_{-\infty}^{+\infty} e^{-\frac{[\alpha-x_0(1+\beta\tau)-\gamma\tau]^2+[x-\alpha(1+\beta\tau)-\gamma\tau]^2}{4\tau D}}\,d\alpha =$$

$$=\frac{1}{(2\sqrt{\pi\tau D})^2}\, e^{-\frac{[x_0(1+\beta\tau)+\gamma\tau]^2+[x-\gamma\tau]^2}{4\tau D}} \int_{-\infty}^{+\infty} e^{-\frac{\alpha^2[1+(1+\beta\tau)^2]-2\alpha\overbrace{[x_0(1+\beta\tau)+\gamma\tau+x(1+\beta\tau)-\gamma\tau(1+\beta\tau)]}^{(x+x_0)(1+\beta\tau)-\gamma\beta\tau^2}}{4\tau D}}\,d\alpha$$

$$e^{-\frac{[x-\gamma\tau]^2+[x_0(1+\beta\tau)+\gamma\tau]^2}{4\tau D}+\frac{[(x+x_0)(1+\beta\tau)-\gamma\beta\tau^2]^2}{[1+(1+\beta\tau)^2]4\tau D}}\sqrt{\frac{\pi\cdot 4\tau D}{1+(1+\beta\tau)^2}}$$

$$e^{-\frac{\cancel{x_0^2(1+\beta\tau)^2}+2\gamma\tau x_0(1+\beta\tau)+2\gamma^2\tau^2+x^2-2x\gamma\tau+x_0^2(1+\beta\tau)^4+2\gamma\tau x_0(1+\beta\tau)^3+2\gamma^2\tau^2(1+\beta\tau)^2}{4\tau D[1+(1+\beta\tau)^2]}}$$

$$+\cancel{x^2(1+\beta\tau)^2}-2x\gamma\tau(1+\beta\tau)^2 - \cancel{(x+x_0)^2(1+\beta\tau)^2}+2(x+x_0)(1+\beta\tau)\gamma\beta\tau^2-\gamma^2\beta^2\tau^4$$

$$2xx_0(1+\beta\tau)^2$$

$$=e^{-\cancel{x-x_0(1+\beta\tau)^2-\gamma\tau(1+\beta\tau)}}$$

$$+2x\gamma\beta^2\tau^3$$

$$+\cancel{2x\gamma\beta\tau^2(1+\beta\tau)}$$

$$x^2-4x\gamma\tau-\cancel{2x\gamma\beta\tau^2}-\cancel{2x\gamma\beta^2\tau^3}+\cancel{2x\gamma\beta\tau^2}+\cancel{2x\gamma\beta^2\tau^3}-2xx_0(1+\beta\tau)^2$$

$$+\cancel{2\gamma\tau x_0+2\gamma\beta\tau^2x_0+2\gamma\tau x_0(1+\beta\tau)^3+2x_0\gamma\beta\tau^2(1+\beta\tau)}+x_0^2(1+\beta\tau)^4$$

$$+4\gamma^2\tau^2+4\beta\gamma^2\tau^3+2\gamma^2\beta^2\tau^4\ \cancel{-\gamma^2\beta^2\tau^4}$$

$$+2\gamma\tau x_0[1+\beta\tau+(1+\beta\tau)^3+\beta\tau(1+\beta\tau)] = (1+\beta\tau)^2+(1+\beta\tau)^3=(1+\beta\tau)^2(2+\beta\tau)$$

$$x-2\gamma\tau+\gamma\beta^2\tau^3-x_0(1+\beta\tau)^2$$

$$\int_{-\infty}^{+\infty} \frac{1}{2\sqrt{\pi D t}}\, \xi^2 e^{-\frac{\xi^2}{4Dt}}\, d\xi = 2Dt$$

$$\frac{1}{2\sqrt{\pi D t}} \int_{l}^{\infty} \xi e^{-\frac{\xi^2}{4Dt}}\, d\xi = 4Dt \int_{\frac{l}{\sqrt{4Dt}}}^{\infty} x e^{-x^2} dx = -4Dt \left. \frac{e^{-x^2}}{2} \right|_{\frac{l}{\sqrt{4Dt}}}^{\infty} = \frac{8Dt \cdot e^{-\frac{l^2}{4Dt}}}{2\sqrt{\pi D t}}$$

$2Dt \gg l^2$

$$\overline{\xi^2} = 2Dt - \frac{4l}{\pi}\sqrt{Dt}\; e^{-\frac{l^2}{4Dt}} + \underbrace{\frac{2l^2}{\sqrt{\pi D t}} \int_{l}^{\infty} e^{-\frac{\xi^2}{4Dt}} d\xi}_{\frac{4l^2}{\sqrt{\pi}} \int_{\frac{l^2}{4Dt}}^{\infty} e^{-x^2} dx}$$

$$\lim_{t=\infty} \overline{\xi^2} = 2Dt - \frac{4l}{\pi}\sqrt{Dt}\; \cancel{\quad} + 2l^2$$

$$\frac{l^2}{4Dt} = \beta^2 \qquad \beta = \frac{l}{2\sqrt{Dt}} \qquad 2\sqrt{Dt} = \frac{l}{\beta}$$

$$\overline{\xi^2} = 2Dt - \frac{2l^2}{\pi\beta} e^{-\beta^2} + \frac{4l^2}{\sqrt{\pi}} \int_{\beta}^{\infty} e^{-x^2} dx$$

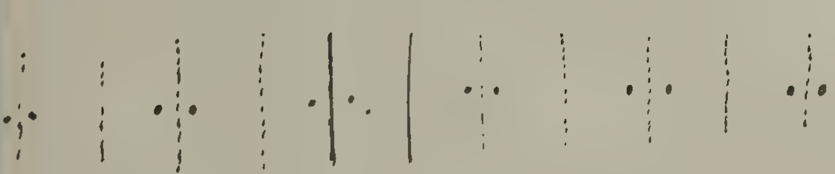
$\xi = \eta - 2l$

$$\int_{-l}^{\infty} \xi e^{-\frac{\xi^2}{4Dt}} d\xi + \underbrace{\int_{-l}^{\infty} \xi e^{-\frac{(2l+\xi)^2}{4Dt}} d\xi}_{+\int_{+l}^{\infty} (\eta - 2l) e^{-\frac{\eta^2}{4Dt}} d\eta}$$

$$\frac{\sqrt{Dt}}{\sqrt{\pi}}\, e^{-\frac{l^2}{4Dt}} - \frac{8l}{8\sqrt{\pi D t}} \int_{\frac{l}{2\sqrt{Dt}}}^{\infty} e^{-x^2} dx$$

$$\frac{l}{2\beta\sqrt{\pi}} e^{-\beta^2} - \frac{2\beta}{\sqrt{\pi}} \int_{\beta}^{\infty} e^{-x^2} dx$$

a



$$e^{-\xi^2} + e^{-(2a+\xi)^2} + e^{-(2a+2b+\xi)^2} + e^{-(4a+2b+\xi)^2} + e^{-(4a+4b+\xi)^2} + \dots$$
$$e^{-(2b-\xi)^2} + e^{-(2a+2b-\xi)^2} + e^{-(2a+4b-\xi)^2} + e^{\cdots} - \dots$$

$$e^{-\xi^2} + e^{-(2\alpha+\xi)^2} + e^{-(4\alpha+\xi)^2} + e^{-(6\alpha+\xi)^2} + \dots$$
$$e^{-(2\alpha-\xi)^2} + e^{-(4\alpha-\xi)^2} + e^{-(6\alpha-\xi)^2} + \dots$$

$2n\alpha + \xi = \eta$

$2n\alpha - \xi = \eta$

$\xi = 2n\alpha - \eta$

$$\int_{-\alpha}^{+\alpha} \xi^2 e^{-(2n\alpha+\xi)^2} d\xi = \int_{(2n-1)\alpha}^{(2n+1)\alpha} (\eta - 2n\alpha)^2 e^{-\eta^2} d\eta = \int \eta^2 e^{-\eta^2} d\eta - 4n\alpha \int \eta e^{-\eta^2} d\eta + 4\alpha^2 n^2 \int e^{-\eta^2} d\eta$$

$$\int_{-\alpha}^{+\alpha} \xi^2 e^{-(2n\alpha-\xi)^2} d\xi = -\int_{(2n+1)\alpha}^{(2n-1)\alpha} (2n\alpha - \eta)^2 e^{-\eta^2} d\eta =$$

$$\overline{\xi^2} = 2\int_0^{\infty} \eta^2 e^{-\eta^2} d\eta - 4\alpha \sum_{n=1}^{\infty} n \int_{(2n-1)\alpha}^{(2n+1)\alpha} \eta e^{-\eta^2} d\eta + 4\alpha^2 \sum_{1} n^2 \int_{(2n-1)\alpha}^{(2n+1)\alpha} e^{-\eta^2} d\eta$$

$$\overline{x-x_0^2} = \frac{1}{2}\sqrt{\frac{\beta}{\pi D(1-e^{-2\beta t})}}\int (x-x_0)^2$$

$$\overline{x^2} = \frac{1}{2}\sqrt{\frac{\beta}{\pi D(1-e^{-2\beta t})}}\;\frac{\int_{-\infty}^{+\infty} x^2 e^{-\frac{\beta(x-x_0e^{-\beta t})^2}{2D[1-e^{-2\beta t}]}}\,dx}{\int e^{- \cdots}\,dx}$$

$$x - x_0 e^{-\beta t} = z$$

$$\frac{\int_{-\infty}^{+\infty}\left[z + x_0 e^{-\beta t}\right]^2 e^{-\gamma z^2}\,dz}{\int_{-\infty}^{+\infty} e^{-\gamma z^2}\,dz} = \overline{z^2} + \cancel{2x_0e^{-\beta t}} + \left[x_0 e^{-\beta t}\right]^2$$

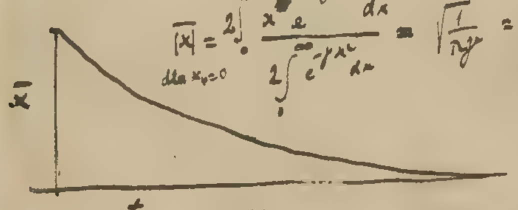
$$= \frac{1}{2\gamma} + \nearrow$$

$$\overline{x^2} = \frac{2D\left[1-e^{-2\beta t}\right]}{2\beta} + x_0^2 e^{-2\beta t}$$

$$\bar{x} = \frac{\int\left[z + x_0 e^{-\beta t}\right] e^{-\gamma z^2}\,dz}{\int e^{-\gamma z^2}\,dz} = x_0 e^{-\beta t}$$

$$\overline{|x|} = \frac{2\int_0^\infty x^2 e^{-\gamma x^2}\,dx}{2\int_0^\infty e^{-\gamma x^2}\,dx} = \sqrt{\frac{1}{\pi\gamma}} = \frac{1}{\sqrt{\frac{\pi\beta}{2D(1-e^{-2\beta t})}}} = \sqrt{\frac{2D(1-e^{-2\beta t})}{\pi\beta}}$$

dla $x_0 = 0$



$$\frac{d\bar{x}}{dt} = -\beta x_0 e^{-\beta t} = -\beta\bar{x} \quad \boxed{\text{relaxacya!}}$$

$$dt = -\frac{1}{\beta}\frac{d\bar{x}}{\bar{x}} \;\Big|\; \cancel{\frac{dt}{\beta x_0}}$$

$$\left\|\frac{d\overline{|x|}}{dt} = \frac{e^{-2\beta t}\,2\beta}{\sqrt{1-e^{-2\beta t}}}\sqrt{\frac{2D}{\pi\beta}}\right|_{t=0} \infty$$

sonst würde man auch $T=0$ aber in Wirklichkeit kann $\frac{dx}{dt}$ nur mittels Abkühlungsgesetz gefunden werden (Ungenauigkeit der ...) das kommt ein wahrer Wert für T

$\beta =$ Sedimentierbarkeit, wenn Abstand $x_0 = 1$

$$= -B\,X_1$$

$$\lim_{t=\infty} \overline{x^2} = \frac{2D}{2\beta} = \frac{RT\,B}{N\,B\,X_1}$$

$$\frac{\overline{x^2}\cdot X_1}{2} = \text{mittlere Arbeit} = \frac{\kappa}{2}$$

$$= \frac{H\theta}{2N}$$

~~sprężystość!~~ sollte $\frac{1}{2}\frac{H\theta}{N}$ sein!

Stimmt! Arbeitet 2β Stellung?

$$dW = \sqrt{\frac{\beta}{2\pi D}}\,e^{-\frac{\beta x^2}{2D}}\,dx = \frac{dt}{T} = \frac{1}{T}\frac{dx}{\beta x}$$

$$T = \frac{\sqrt{\pi D}\;e^{+\frac{\beta x^2}{2D}}}{x\beta\sqrt{\beta}}$$

$$T \gtrless \sqrt{2\pi\frac{\bar{x}^2}{x^2}}\,\frac{1}{\beta}\,e^{\frac{x^2}{2\bar{x}^2}}$$

$$W_1(x, x_0, \tau) = \frac{1}{2\sqrt{\pi\tau D}}\,e^{-\frac{[x - x_0(1+\beta\tau)]^2}{4\tau D}}$$

można β ujemne!

$$W_2(x, x_0, \tau) = \frac{1}{2\sqrt{\pi\tau D}\sqrt{1+(1+\beta\tau)^2}}\,e^{-\frac{\left[x - x_0(1+\beta\tau)^2\right]^2}{4\tau D[1+(1+\beta\tau)^2]}}$$

$$W_3(x, x_0, \tau) = \frac{1}{2\sqrt{\pi\tau D}\sqrt{1+(1+\beta\tau)^2[1+(1+\beta\tau)^2]}}\,e^{-\frac{\left[x - x_0(1+\beta\tau)^3\right]^2}{4\tau D[1+(1+\beta\tau)^2[1+(1+\beta\tau)^2]]}}$$

$$W_n(x, x_0, n\tau) = \frac{1}{2\sqrt{\pi\tau D}}\,\frac{1}{\sqrt{\underbrace{1+(1+\beta\tau)^2+(1+\beta\tau)^4+\cdots(1+\beta\tau)^{2(n-1)}}_{\varphi}}}\,e^{-\frac{\left[x - x_0(1+\beta\tau)^n\right]^2}{4\tau D\underbrace{[1+(1+\beta\tau)^2+\cdots\cdots(1+\beta\tau)^{2(n-1)}]}_{=\varphi_n}}}$$

$$\varphi_n = \frac{(1-\beta\tau)^{2n}-1}{-\beta\tau} = 2n + \frac{2n(2n-1)}{2}\beta\tau + \cdots$$

$$= \frac{s^{n+1}-1}{s-1} = \frac{(1-\beta\tau)^{2n+2}-1}{(1+\beta\tau)^2-1} = -\frac{e^{-2n\beta\tau}-1}{2\beta\tau} = \frac{1-e^{-2\beta t}}{2\beta\tau}$$

$$(1-\beta\tau)^n = (1-\beta\tau)^{-n\beta\tau\cdot\frac{1}{\beta\tau}} = e^{-n\beta\tau}$$

$$s = 1+q^n$$

$$1+q+q^2+\cdots q^{n-1} = s$$

$$q^n - 1 = s(q-1)$$

$$s = \frac{q^n-1}{q-1}$$

$$W_n(x, x_0, t) = \cancel{\frac{1}{2\sqrt{\pi\tau D}}}\,\frac{1}{\sqrt{2n}}\,e^{-\frac{\left[x - x_0 e^{-n\beta\tau}\right]^2}{4D\tau\cdot 2n}}$$

$$= \frac{1}{2\sqrt{\pi t D}}\,e^{-\frac{\left[x - x_0 e^{-\beta t}\right]^2}{8tD}}$$

$$\left\|\frac{e^{-\frac{\left[x-x_0e^{-n\beta\tau}\right]^2 2\beta\tau}{4D\tau(1-e^{-2\beta t})}}}{2\sqrt{\pi\tau D}\cdot\frac{(1-e^{-2\beta t})}{2\beta\tau}}\right.$$

$$\lim_{t=0} S = \lg\sqrt{\frac{\beta}{\pi D}} - \frac{\beta}{2D}x^2$$

$$W(x, x_0, t) = \frac{\sqrt{2\beta}}{2\sqrt{\pi D(1-e^{-2\beta t})}}\,e^{-\frac{\left[x - x_0 e^{-\beta t}\right]^2 2\beta}{4D[1-e^{-2\beta t}]}} = \sqrt{\frac{\beta}{2\pi D(1-e^{-2\beta t})}}\,e^{-\frac{\beta\left[x-x_0e^{-\beta t}\right]^2}{2D[1-e^{-2\beta t}]}}$$

określa ruch Browna przy istnieniu siły sprężystej działającej ku środkowi współrzędnych, jeżeli punkt za czas $t=0$ wyszedł z pozycji x_0

Samo odbicie v bliskości ściany

$\cancel{W(x,x_0)}$

$$W(x, x_0, \tau)\,dx = \frac{1}{2\sqrt{\pi\tau D}}\left[e^{-\frac{(x-x_0)^2}{4\tau D}} + e^{-\frac{(x+x_0)^2}{4\tau D}}\right]dx$$

$$W(x, x_0, \tau)\,dx = \frac{1}{2\sqrt{\pi\tau D}}\left[e^{-\frac{(x-x_0+\gamma\tau)^2}{4\tau D}} + e^{-\frac{(x+x_0-\gamma\tau)^2}{4\tau D}}\right]dx$$

$$W(x, x_0, 2\tau)\,dx = dx\int_{\alpha=0}^{\infty} W(\alpha, x_0, \tau)\,d\alpha\; W(x, \alpha, \tau)$$

$$\int_0^\infty e^{-\frac{(\alpha-x_0+\gamma\tau)^2}{4\tau D}-\frac{(x-\alpha+\gamma\tau)^2}{4\tau D}}\,d\alpha + \int e^{\frac{-(\alpha-x_0+\gamma\tau)^2-(x+\alpha-\gamma\tau)^2}{4\tau D}}\,d\alpha +$$

$$+\int e^{\frac{-(\alpha+x_0-\gamma\tau)^2-(x+\alpha-\gamma\tau)^2}{4\tau D}}\,d\alpha + \int_0^\infty e^{\frac{-(\alpha+x_0-\gamma\tau)^2-(x-\alpha+\gamma\tau)^2}{4\tau D}}\,d\alpha$$

$$\int e^{-\left\{2\alpha^2+2\alpha(-x_0+\gamma\tau+x+\gamma\tau)+(x_0-\gamma\tau)^2+(x+\gamma\tau)^2\right\}}$$

$$\frac{m}{2}\frac{d^2}{dt^2}(x^2) + \frac{1}{2\beta}\frac{d(x^2)}{dt} = \mathfrak{F}x - \gamma x + m\left(\frac{dx}{dt}\right)^2$$

$$\frac{m}{2}\frac{d(x^2)}{dt} + \frac{x^2}{2\beta} = \int x\mathfrak{F}\,dt - \gamma\int x\,dt + 2t\bar{L}$$

A w razie siły sprężystej:

$$\frac{m}{2}\frac{d(x^2)}{dt} + \frac{x^2}{2\beta} = \int x\mathfrak{F}\,dt - \gamma\int x^2\,dt + 2t\bar{L}$$

Więc $\frac{m}{2}\frac{d\bar{x^2}}{dt} + \frac{\bar{x^2}}{2\beta} + \gamma\int\bar{x^2}\,dt = 2t\bar{L}$

Równanie czyniące zadość dla $\bar{x^2}$. Czy się spełnia przy podstawieniu w równanie?

$$\bar{x^2} = \cancel{kD}\left[1-e^{-2\beta t}\right] + x_0^2 e^{-4\beta t}$$

$$= k\left[1-e^{-2\beta t}\right] + x_0^2 e^{-4\beta t}$$

$D = k\beta$

$$\int\bar{x^2}\,dt = k\left[t + \frac{(e^{-2\beta t}-1)}{2\beta}\right] - \frac{x_0^2}{4\beta}(e^{-4\beta t}-1)$$

$$\frac{d}{dt}(\bar{x^2}) = -2\beta k e^{-2\beta t} - 4\beta x_0^2 e^{-4\beta t}$$

$$\frac{d^2}{dt^2}(\bar{x^2}) = -4\beta^2 k e^{-2\beta t} - 16\beta^2 x_0^2 e^{-4\beta t}$$

$$\frac{m}{2}\left[-2\beta k e^{-2\beta t} - 4\beta x_0^2 e^{-4\beta t}\right] + \frac{k}{2\beta}\left[1-e^{-2\beta t}\right] + \frac{x_0^2}{2\beta}e^{-4\beta t} + \gamma k t + \frac{k(e^{-2\gamma t}-1)}{2}$$

$$-\frac{x_0^2}{4}(e^{-4\gamma t}-1) = 2t\,k\,\|$$

Przy istnieniu ciężkości oprócz siły sprężystej

$$W_{(\xi, x, \tau)}\, d\xi = A\, e^{-\frac{(\xi - \beta x \tau - \gamma\tau)^2}{4\tau D}}\, d\xi$$

$$= \frac{1}{2\sqrt{\pi\tau D}}\, e^{-\frac{[x - x_0(1+\beta\tau) - \gamma\tau]^2}{4\tau D}}\, dx$$

$$= \frac{1}{2\sqrt{\pi\tau D}}\, e^{-\frac{[x - x_0 - (x_0\beta + \gamma)\tau]}{4\tau D}}\, dx$$

$$e^{-\frac{1}{4\tau D}\left\{[\alpha - x_0(1+\beta\tau) - \gamma\tau]^2 + [x - \alpha(1+\beta\tau) - \gamma\tau]^2\right\}}$$

$$= e^{-\frac{1}{4\tau D}\left\{[x_0(1+\beta\tau) + \gamma\tau]^2 + [x - \gamma\tau]^2\right\} - \alpha^2[1 + (1+\beta\tau)^2] - 2\alpha\,[x_0(1+\beta\tau) + \cancel{\gamma\tau} + x(1+\beta\tau) \underbrace{-\gamma\beta\tau^2}]}$$

$$= e^{-\frac{1}{4\tau D}\left\{\cancel{\frac{[x - x_0(1+\beta\tau)^2]^2 + 2\gamma\tau[x_0(1+\beta\tau) - x] + 2\gamma\tau^2}{1 + (1+\beta\tau)^2}}\right\}}$$

$$= e^{-\frac{1}{4\tau D}}\left\{\cancel{x - x_0(1+\beta\tau)}\right.$$

$$+\gamma^2\beta^2\tau^4$$

$$-2\gamma\beta\tau^2 x_0(1+\beta\tau) - 2\gamma\beta\tau^2 x(1+\beta\tau)$$

$$\frac{\left\{[x_0(1+\beta\tau) + \gamma\tau]^2 + [x - \gamma\tau]^2\right\}[1 + (1+\beta\tau)^2] + [x_0 + x(1+\beta\tau)]^2(1+\beta\tau)^2}{1 + (1+\beta\tau)^2}$$

$$= \frac{[x - x_0(1+\beta\tau)^2]^2}{1 + (1+\beta\tau)^2} + \cancel{2\gamma\tau[x_0(1+\beta\tau) - x] + 2\gamma\tau^2}$$

$$W_1 = e^{-\frac{[x - x_0(1+\beta\tau)]^2 - 2\gamma\tau[x - x_0(1+\beta\tau)] + \gamma^2\tau^2}{}}$$

$$W_2 = e^{-\frac{x x_0(1+\beta\tau)^2}{1 + (1+\beta\tau)^2} - 2\gamma\tau[x - x_0(1+\beta\tau)] + 2\gamma^2\tau^2}$$

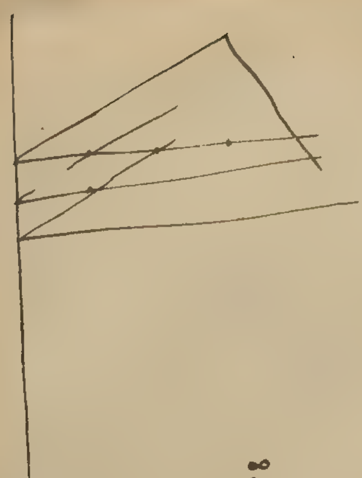
$$x_0^2(1+\beta\tau)^2 \cancel{\gamma^2\tau^2} + 2\gamma\tau x_0(1+\beta\tau) + x^2 - 4x\gamma\tau - 2x\beta\gamma\tau^2 - \cancel{2x\gamma\beta\tau^2}$$

$$+ x_0^2(1+\beta\tau)^4 + \cancel{2\gamma\tau(1+\beta\tau)^2} + 2\gamma\tau x_0(1+\beta\tau)^3 + x^2(1+\beta\tau)^2 - \cancel{2x\gamma\tau(1+\beta\tau)^2}$$

$$\cancel{\left\{x_0^2(1+\beta\tau)^2 + x^2(1+\beta\tau)^4 + 2 x x_0 (1+\beta\tau)^3 - 2\gamma\beta\tau^2 x_0(1+\beta\tau) - 2\gamma\beta\tau^2 x(1+\beta\tau) + \gamma^2\beta^2\tau^4\right\}}$$

$$+ x_0^2(1+\beta\tau)^2 + x^2(1+\beta\tau)^2 + \cancel{\gamma^2\tau^2} + 2 x x_0 (1+\beta\tau)^2 - 2 x_0\gamma\beta\tau^2(1+\beta\tau) - \cancel{2x\gamma\beta\tau^2(1+\beta\tau)}$$

$$+ 4\gamma^2\tau^2 + 4\gamma^2\beta\tau^3 + \gamma^2\beta^2\tau^4$$



$\cancel{\bar W(x,\alpha) = \frac{1}{2\lambda} \int d\alpha}$

$$\bar W_n(\alpha) = \frac{1}{2\lambda}\int_{x-\lambda+\varepsilon}^{x+\lambda+\varepsilon} \bar W_{n-1}(\alpha, x_0)\, d\alpha \qquad \text{für } \alpha > \lambda-\varepsilon$$

$$\bar W_n(\alpha) = \frac{1}{2\lambda}\int_0^{x+\lambda+\varepsilon} \bar W_{n-1}(\alpha, x_0)\, d\alpha + \int_0^{\lambda-\varepsilon-x} \bar W_{n-1}(\alpha, x_0)\, d\alpha \qquad 0 > \alpha < \lambda-\varepsilon$$

$$= \frac{1}{2\lambda}\left[\int_{\lambda-\varepsilon+x}^{x+\lambda+\varepsilon} \cdots\cdots + 2\int_0^{\lambda-\varepsilon-x} \cdots\cdots\right]$$

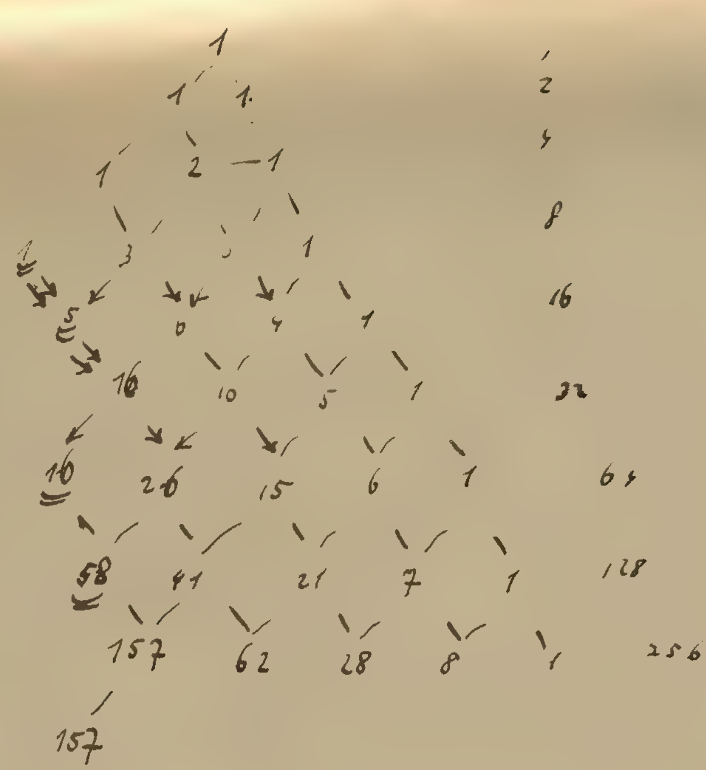
$$\bar W_1(\alpha, x_0) = \frac{1}{\pi}\int_0^\infty \frac{\sin \varrho\lambda}{\varrho\lambda} \cos\varrho(\alpha - x_0 + \varepsilon)\, d\varrho \qquad \text{für } \alpha > \lambda-\varepsilon$$

$$\bar W_2(\alpha, x_0) = \frac{1}{\pi}\int_0^\infty \left(\frac{\sin\varrho\lambda}{\varrho\lambda}\right)^2 \cos\varrho(\alpha - x_0 + 2\varepsilon)\, d\varrho$$

$$\bar W_3(\alpha, x_0) = \frac{1}{\pi}\int_0^\infty \left(\frac{\sin\varrho\lambda}{\varrho\lambda}\right)^3 \cos\varrho(\alpha - x_0 + 3\varepsilon)\, d\varrho$$

$$\bar W_2(\alpha, x_0) = \frac{1}{2\lambda}\left\{\frac{1}{\pi}\int_0^\infty \frac{\sin\varrho\lambda}{\varrho\lambda}\, d\varrho \left[\int_0^{x+\lambda+\varepsilon} \cos\varrho(\alpha - x_0 + \varepsilon)\, d\alpha + \int_0^{\lambda-\varepsilon-x} \cos\varrho(\alpha - x_0 + \varepsilon)\, d\alpha\right]\right\} \qquad 0 < \alpha < \lambda-\varepsilon$$

$$= \frac{1}{2\pi}\int_0^\infty \frac{\sin\varrho\lambda}{(\varrho\lambda)^2}\left[\sin\varrho(\alpha - x_0 + \lambda + 2\varepsilon) + \sin\varrho(x_0 - \varepsilon) + \sin\varrho(\lambda - x_0 - x) + \sin\varrho(x_0 - \varepsilon)\right]$$



$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} + \gamma\frac{\partial c}{\partial x} \qquad \frac{\partial c}{\partial x} = 0 \Big|^{x=0} \quad \text{also} \int_0^\infty c\,dx = \text{const}$$

$$\int e^{-\frac{x^2}{Dt}}$$

$$\frac{1}{t} \quad \ldots \left[\left(\frac{x}{t}-\alpha\right)^2 + \frac{1}{t}\right] + \gamma\left(\frac{x}{t}-\alpha\right)$$

$$\frac{e^{-\frac{x^2}{t} - \alpha x}}{\sqrt{t}} \qquad 2\alpha D = \gamma$$

$D\alpha^2 = \alpha\gamma$

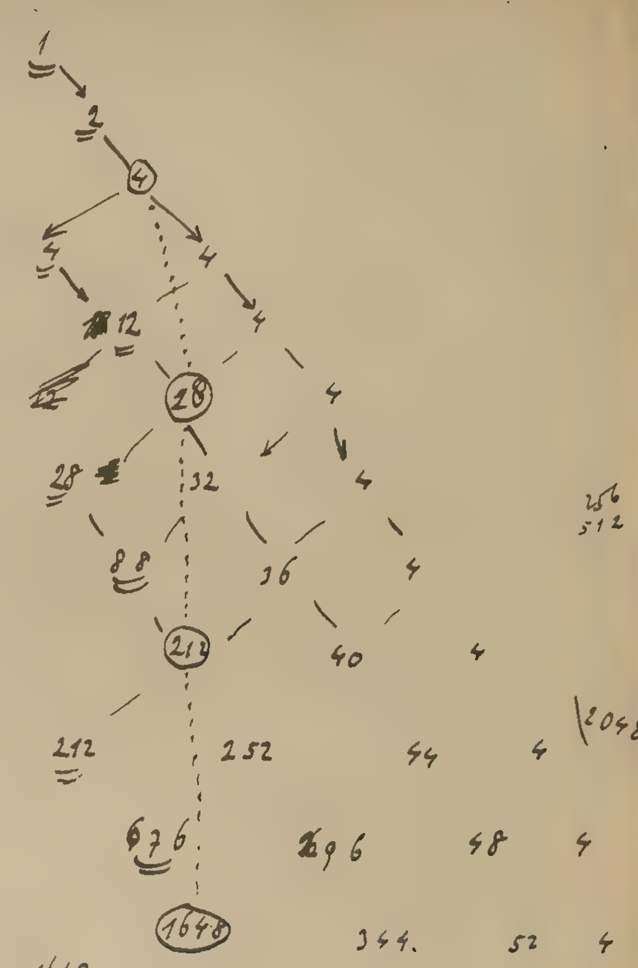
$$\cancel{c = u e^{-\alpha x} + \ldots}$$

$$\cancel{\frac{\partial u}{\partial t} e^{-\alpha x} = \gamma\left[\frac{\partial u}{\partial x} - \alpha u\right]e^{-\alpha x} + D\left[\frac{\partial^2 u}{\partial x^2} - 2\alpha\frac{\partial u}{\partial x} + \alpha^2 u\right]e^{-\alpha x}}$$

$$\gamma = 2\alpha D$$
$$\gamma\alpha = \alpha D$$

$$1 \quad \frac{4}{8} \quad \frac{28}{64} \quad \frac{212}{512} \quad \frac{1648}{4096}$$

$$1 \cdot \frac{1}{2} \cdot \frac{7}{16} \cdot \frac{53}{128} \cdot \frac{103}{256}$$



$$\int_0^\infty W(\alpha, x_0, \tau)\, W(x, \alpha, \tau)\, d\alpha =$$

$$W(x, x_0, n\tau) = \int_0^\infty W(\alpha, x_0, (n-1)\tau)\, W(x, \alpha, \tau)\, d\alpha$$

$$= \cancel{\int \left[W(\alpha, x_0, n\tau) - \frac{\partial W}{\partial \tau}(\alpha, x_0, n\tau)\, W(x,\alpha,\tau) \right] W(x,\alpha,\tau)\, d\alpha}$$

$$= \underbrace{W(x, \infty, \tau)}_{=0} \int_0^\infty \underbrace{W(\alpha, x_0, (n-1)\tau)\, d\alpha}_{=1} - \cancel{W(x, 0, \tau) \int_0^0} - \int_0^\infty d\alpha\, \frac{\partial W(x,\alpha,\tau)}{\partial \alpha} \int_0^\alpha W(\alpha, x_0, (n-1)\tau)\, d\alpha$$

$$\int_0^\infty e^{-\frac{(x - x_0 + \gamma\tau)^2}{4\tau D}} dx + B \int_0^\infty e^{-\frac{(x + x_0 + \gamma\tau)^2}{4\tau D}} dx = \int_0^\infty e^{-\frac{(x-x_0)^2}{4\tau D}} - \gamma\tau$$

$$e^{-\frac{(x-x_0)^2}{4\tau D} - \frac{\gamma}{2D}(x - x_0)}$$

$$W_n(x, x_0) = \int_0^\infty W_{n-1}(\alpha, x_0)\, W_1(x, \alpha)\, d\alpha$$

$$\cancel{W_2(x, x_0) = \int_0^\infty e^{-\alpha^2 + 2\alpha(x + x_0) - (x_0 - \gamma\tau)^2 - (x + \gamma\tau)^2} d\alpha = e^{-x^2 - x_0^2 - 2\gamma\tau(x - x_0)} \frac{\int e^{-\alpha^2 + 2\alpha(x + x_0)} d\alpha}{-(x-\beta+\gamma\tau)^2}}$$

$$\cancel{W_2(\beta, x_0) = \int e^{-\alpha^2 + 2\alpha(\beta + x_0) - (x_0 - \gamma\tau)^2 - (\beta + \gamma\tau)^2} d\alpha \qquad W_1(x, \beta) = e}$$

$$\cancel{W_3(x, x_0) = \int d\beta\, d\alpha\; e^{-\alpha^2 + 2\alpha(\beta + x_0) - (x_0 - \gamma\tau)^2 - (\beta + \gamma\tau)^2 - (x - \beta + \gamma\tau)^2}}$$

$$p_1(z) = \frac{1}{\pi} \int_0^\infty d\varrho \int_{-\infty}^{+\infty} p_1(\beta) \cos\varrho(z - \beta)\, d\beta$$

$$p_1(z, x_0) = \frac{1}{\pi} \int_0^\infty d\varrho \int_0^{+\infty} \left[e^{-(\beta - x_0 + \gamma\tau)^2} + e^{-(\beta + x_0 - \gamma\tau)^2} \right] \cos\varrho(z - \beta)\, d\beta$$

$$p(x, z) = \frac{1}{\pi} \int_0^\infty d\varrho \int_0^\infty \left[e^{-(\alpha - z + \gamma\tau)^2} + e^{-(\alpha + z - \gamma\tau)^2} \right] \cos\varrho(x - \alpha)\, d\alpha$$

$$\cancel{\int e^{-\frac{(x-x_0)^2}{4\tau D} - \frac{\gamma}{2D}(x - x_0)} dx} \quad A\; e^{-\frac{\gamma}{2D}x_0 + \frac{(\gamma\tau)^2}{4\tau D}} \int_0^\infty e^{-\frac{(x - x_0 + \gamma\tau)^2}{4\tau D}} dx = 1$$

$$A = \frac{e^{\frac{\gamma}{2D}x_0 - \frac{(\gamma\tau)^2}{4\tau D}}}{\int_0^\infty \cdots}$$

Ist Lösung:
$$W(x, x_0, \tau) = \frac{e^{-\frac{(x - x_0 + \gamma\tau)^2}{4\tau D}}}{\int_0^\infty e^{-\frac{(x - x_0 + \gamma\tau)^2}{4\tau D}} dx}$$

Lösung stationär $\lim_{\tau=\infty} = e^{-\frac{x}{\cdot}}$

$$W(x, x_0, \tau)\,dx = A\left[e^{-\frac{(x-x_0+\gamma\tau)^2}{4\tau D}}dx + e^{-\frac{(x+x_0-\gamma\tau)^2}{4\tau D}}dx\right]$$

$$\int_0^\infty e^{-\frac{(x-x_0)^2}{\mu}}dx + \int_0^\infty e^{-\frac{(x+x_0)^2}{\mu}}dx$$

$$\int_{x_0}^\infty e^{-\frac{\xi^2}{\mu}}d\xi + \int_{-x_0}^\infty e^{-\frac{\xi^2}{\mu}}\frac{d\xi}{=-\int_{+x_0}^{-\infty} e^{\xi}d\xi}$$

$$\int_0 e^{-\frac{\xi^2}{\mu}}d\xi$$

$$W(x, x_0, 2\tau)\,dx = \int_0^\infty W(\alpha, x_0, \tau)\,dx\; W(x, \alpha, \tau)$$

$$= A^2\int_0^\infty e^{-(\alpha-x_0+\gamma\tau)^2-(x-\alpha+\gamma\tau)^2} + e^{-(\alpha-x_0-\gamma\tau)^2-(x+\alpha-\gamma\tau)^2} + e^{-(\alpha+x_0-\gamma\tau)^2-(x-\alpha+\gamma\tau)^2} + e^{-(x+\alpha-\gamma\tau)^2-(\alpha+x_0-\gamma\tau)^2}\,d\alpha$$

$$\int_0^\infty e^{-\alpha^2+2\alpha(x+x_0)-(x_0-\gamma\tau)^2-(x+\gamma\tau)^2}d\alpha + \int e^{-\alpha^2+2\alpha(x_0-\cancel{x})-(x_0-\gamma\tau)^2-(x-\gamma\tau)^2}d\alpha +$$

$$+\int_0^\infty e^{-\alpha^2+4\alpha\gamma\tau-2\alpha(x_0-x)-(x_0-\gamma\tau)^2-(x+\gamma\tau)^2}d\alpha + \int_0^\infty e^{-\alpha^2+4\alpha\gamma\tau-2\alpha(x+x_0)-(x-\gamma\tau)^2-(x_0-\gamma\tau)^2}d\alpha$$

W razie małego τ:

$$e^{-\frac{(x-x_0)^2}{4\tau D}-\frac{\gamma}{2D}(x-x_0)-\frac{\gamma^2\tau}{4D}}$$

$$= e^{-\frac{(x-x_0)^2}{4\tau D}}\left[1-\frac{\gamma}{2D}(x-x_0)+\frac{\gamma^2}{4D^2}(x-x_0)^2-\gamma^2\frac{\tau}{4D}\right] +$$

$$+ e^{-\frac{(x+x_0)^2}{4\tau D}}\left[1+\frac{\gamma}{2D}(x+x_0)+\frac{\gamma^2}{4D^2}(x+x_0)^2-\gamma^2\frac{\tau}{4D}\right]$$

$$\int_0^\infty e^{-\frac{(\alpha-x_0)^2}{4\tau D}}\left[1-\frac{\gamma}{2D}(\alpha-x_0)+\frac{\gamma^2}{4D^2}(\alpha-x_0)^2-\frac{\gamma^2\tau}{4D}\right]e^{-\frac{(x-\alpha)^2}{4\tau D}}\left[1-\frac{\gamma}{2D}(x-\alpha)+\frac{\gamma^2}{4D^2}(x-\alpha)^2-\frac{\gamma^2\tau}{4D}\right]$$

$$\int_0^\infty e^{-\alpha^2+2\alpha\beta}d\alpha = e^{+\beta^2}\int_0^\infty e^{-(\alpha-\beta)^2}d\alpha = e^{\beta^2}\int_\beta^\infty e^{-x^2}dx \qquad \begin{matrix}\alpha-\beta=x\\ \alpha=x+\beta\end{matrix}$$

$$e^{(x+x_0)^2-(x_0-\gamma\tau)^2-(x+\gamma\tau)^2}\int_{x+x_0}^\infty \cdots + e^{(x_0-x)^2-(x_0-\gamma\tau)^2-(x-\gamma\tau)^2}\int_{x_0-x}^\infty \cdots$$

$$e^{2xx_0+2\gamma\tau(x_0-x)-2\gamma^2\tau^2}\int_{x+x_0}^\infty \cdots + e^{-2xx_0+2\gamma\tau(x_0+x)-2\gamma^2\tau^2}\int_{x_0-x}^\infty \cdots$$

$$\Bigg\| \; e^{(x+\gamma\tau)(x_0-\gamma\tau)}$$

$$-(x-\gamma\tau)(x_0-\gamma\tau)$$

$$-(x_0-\gamma\tau)(x+\gamma\tau)$$

$$(x-\gamma\tau)(x_0-\gamma\tau)$$

$$+e^{(2\gamma\tau-x_0+x)^2-(x_0-\gamma\tau)^2-(x+\gamma\tau)^2}\int_{x-x_0+2\gamma\tau}^\infty + e^{(2\gamma\tau-x-x_0)^2-(x-\gamma\tau)^2-(x_0-\gamma\tau)^2}\int_{2\gamma\tau-x-x_0}$$

$$\underbrace{\int_{-\infty}^{x_0-x-2\gamma\tau}}\qquad\qquad \int_{-\infty}^{x_0+x-2\gamma\tau}$$

$$e^{2\gamma^2\tau^2-2\gamma\tau(x_0-x)-2xx_0}\left[\int_{-\infty}^{x_0-x}e^{-\alpha^2}d\alpha - 2\gamma\tau\, e^{-(x_0-x)^2}\right] + e^{2\gamma^2\tau^2-2\gamma\tau(x+x_0)+2xx_0}\left[\int_{-\infty}^{x_0+x} - 2\gamma\tau\, e^{-(x_0+x)^2}\right]$$

$$W_2(x,x_0) = \frac{4c^2}{\pi^2}\int_{-\infty}^{+\infty}\Big[\frac{dq}{q}\sin q(\alpha - x_0+\varepsilon)\frac{dp}{p}\sin p(x-\alpha+\varepsilon)\cos q\lambda\cos p\lambda$$

$$+\frac{dq\,dp}{p\cdot q}\sin q\left(\alpha+x_0\tfrac{\lambda-\varepsilon}{\lambda+\varepsilon}+\varepsilon\right)\sin p(x-\alpha+\varepsilon)\cos q\lambda\cos p\lambda\Big]$$

$$= \frac{2c^2}{\pi}\int_{\alpha-\lambda-\varepsilon}^{\alpha+\lambda-\varepsilon} d\alpha\int_0^\infty \frac{dq}{q}\left[\sin q(x-\alpha+\varepsilon)+\sin q\left(\alpha+\alpha\tfrac{\lambda-\varepsilon}{\lambda+\varepsilon}+\varepsilon\right)\right]\cos q\lambda$$

$$= \frac{2c^2}{\pi}\int^\infty \frac{dq}{q}\cos q\lambda\ \cos q(x$$

$$W_2(x,x_0) = \frac{1}{2\lambda}\int_{-\infty}^{+\infty} d\alpha\int_{x_0-\lambda+\varepsilon}^{x_0+\lambda+\varepsilon}\frac{2}{\pi}\int_0^\infty\frac{dq}{q\lambda}\sin q\lambda\left[\cos q(\alpha-x_0+\varepsilon)+\cos q\left(\alpha+x_0\frac{\lambda-\varepsilon}{\lambda+\varepsilon}+\varepsilon\right)\right]$$

$$= \frac{2}{\pi}\int_0^\infty dq\,\frac{\sin q\lambda}{2q^2\lambda^2}\left[\sin q(x+\lambda+\varepsilon-x_0+\varepsilon)-\sin q(x-\lambda+\varepsilon-x_0+\varepsilon)\right.$$

$$=\left[\sin q(x-x_0+2\varepsilon+\lambda)-\sin q(x-x_0+2\varepsilon-\lambda)\right]$$

$$=2\cos q(x-x_0+2\varepsilon)\sin q\lambda$$

$$= \frac{1}{\pi}\int dq\,\frac{\sin^2 q\lambda}{(q\lambda)^2}\left[\cos q(x-x_0+2\varepsilon)+\cos q\left(x+x_0\frac{\lambda-\varepsilon}{\lambda+\varepsilon}+2\varepsilon\right)\right]$$

$$W_n(x,x_0) = \frac{1}{\pi}\int_0^\infty dq\left(\frac{\sin q\lambda}{q\lambda}\right)^n\left[\cos q(x-x_0+n\varepsilon)+\cos q\left(x+x_0\frac{\lambda-\varepsilon}{\lambda+\varepsilon}+n\varepsilon\right)\right]$$

$$= e^{-\frac{n q^2\lambda^2}{6}}$$

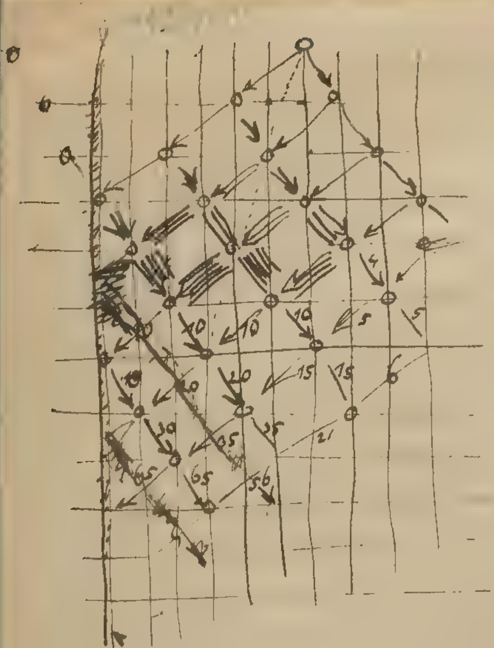
$$\frac{(x-x_0+n\varepsilon)^2}{n\lambda^2}$$

$n\lambda = ct$

$c\lambda = D$

$$\cdots\left\{e^{-\frac{(x-x_0+\gamma t)^2}{4Dt}}+e^{-\frac{(x+x_0 f(t)+\gamma t)^2}{4Dt}}\right\}$$

$$\frac{\lambda-\varepsilon}{\lambda+\varepsilon}=\frac{c-\gamma}{c+\gamma}$$



$$W_n(\alpha, x_0) = \int_0^\infty W_{n-1}(\alpha, x_0)\, W(x, \alpha)\, d\alpha$$

$$W_1(x, x_0) = c \Big|_{x = x_0 - \lambda - \varepsilon}^{x = x_0 + \lambda - \varepsilon} = \frac{1}{2\lambda} \qquad \text{pod warunkiem: } x_0 > \lambda + \varepsilon$$

$$W_1(x, x_0) = 2c \Big|_{x=0}^{x=}$$

Uzupełniając sobie po drugiej stronie ściany obraz domniemany widzimy, że zastępuje zupełnie efekt ściany, zatem otrzymamy całkiem ogólnie będzie: ~~$W_1(x, x_0) = c \Big|_{x = x_0 - \lambda + \varepsilon}^{x = x_0 + \lambda - \varepsilon} \Big|_{x = x_0 - \lambda - \varepsilon}^{x = x_0 + \lambda + \varepsilon}$~~

nie prawda! bo wtedy byłyby punkty wyjścia z prawej strony ku lewej! nie byłoby odbicia zupełnego!

$$W(x, \alpha) = c \Big|_{x = \alpha - \lambda - \varepsilon}^{x = \alpha + \lambda - \varepsilon}$$

$$W_0(x, x_0) = 1 \qquad \frac{x = -x_0 \frac{\lambda - \varepsilon}{\lambda + \varepsilon}}{x = x_0}$$

$$W_1(x, x_0) = c = \frac{1}{2\lambda} \Big|_{x = x_0 - \lambda - \varepsilon}^{x = x_0 + \lambda - \varepsilon} \qquad \Big|_{x = -x_0 \frac{\lambda - \varepsilon}{\lambda + \varepsilon} - \lambda - \varepsilon}^{x = -x_0 \frac{\lambda - \varepsilon}{\lambda + \varepsilon} + \lambda - \varepsilon}$$

$$= \frac{1}{\pi} \int_0^\infty d\varrho \int_{-\infty}^{+\infty} W_1(z) \cos\varrho(x - z)\, dz = \frac{c}{\pi} \int d\varrho \left\{ \int_{x_0 - \lambda - \varepsilon}^{x_0 + \lambda - \varepsilon} \cos\varrho(x - z)\, dz + \int_{-x_0 \frac{\lambda - \varepsilon}{\lambda + \varepsilon} - \lambda - \varepsilon} \cos\varrho(x - z)\, dz \right\}$$

$$W_1(x, x_0) = -\frac{c}{\pi} \int \frac{d\varrho}{\varrho} \left[\sin\varrho(x - x_0 - \lambda + \varepsilon) - \sin\varrho(x - x_0 + \lambda + \varepsilon) + \sin\varrho\left(x + x_0 \frac{\lambda - \varepsilon}{\lambda + \varepsilon} - \lambda + \varepsilon\right) - \sin\varrho\left(x + x_0 \frac{\lambda - \varepsilon}{\lambda + \varepsilon} + \lambda + \varepsilon\right) \right]$$

$$W(x, \alpha) = -\frac{c}{\pi} \int \frac{d\varrho}{\varrho} \left[\sin\varrho(x - \alpha - \lambda + \varepsilon) - \sin\varrho(x - \alpha + \lambda + \varepsilon) \right]$$

$$W_1(x, x_0) = +2\frac{c}{\pi} \int \frac{d\varrho}{\varrho} \left[\cos\varrho(x - x_0 + \varepsilon) + \cos\varrho\left(x + x_0 \frac{\lambda - \varepsilon}{\lambda + \varepsilon} + \varepsilon\right) \right] \sin\varrho\lambda$$

$$W(x, \alpha) = 2\frac{c}{\pi} \int \frac{d\varrho}{\varrho} \cos\varrho(x - \alpha + \varepsilon) \sin\varrho\lambda = c \Big|_{x = \alpha - \lambda - \varepsilon}^{x = \alpha + \lambda - \varepsilon}$$

$$W_1(x, x_0) = \frac{1}{\pi} \int_0^\infty \frac{\sin\varrho\lambda}{\varrho\lambda} \left[\cos\varrho(x - x_0 + \varepsilon) + \cos\varrho\left(x + x_0 \frac{\lambda - \varepsilon}{\lambda + \varepsilon} + \varepsilon\right) \right]$$

[illegible]

[illegible]

∞

$$\delta W = 4\pi\rho m\, r^2 f(r)\, dr$$

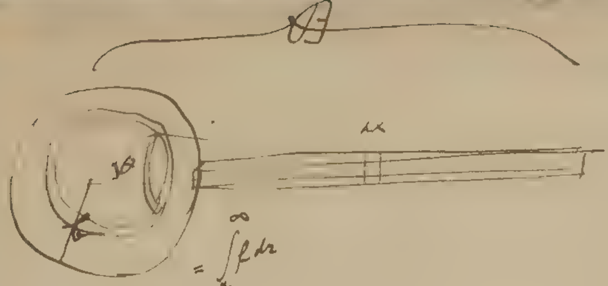
$$W = 4\pi\rho m \int_0 r^2 f(r)\, dr$$

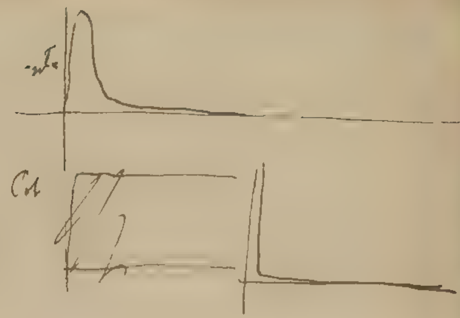
folbro!

$$I = \int_v p\,dv + \underbrace{2\pi\rho \underbrace{\int_0^\infty r^2 f(r)\, dr}_{= -\int_0^a x \frac{d\varphi}{dx}\, dx = \int_\gamma^\infty \varphi(x)\, dx = \frac{a}{2\pi}}}_{= a\rho}$$

po $\frac{v}{g}$. $\quad p(v_1 - v_2)$

$$\bar{I} = \int_v p\,dv + 2\rho$$





$= \int_r^\infty f\,dr$

$m m' \chi(r)$ = Potenzial $\qquad - m m' \chi(r)$ ...

$-2\pi\rho^2\, d\sigma\, dx\, u^2 \sin\vartheta\, du\, d\vartheta\ \frac{d\chi(r)}{dx}$

$p_i\, d\sigma =$

$dA = 2\pi\rho^2\, d\sigma \int_0^b u\,du \int_b^\beta dx\, \frac{d}{dx} \underbrace{\int_0^\pi \chi(r)\, u \sin\vartheta\, d\vartheta}_{= \frac{1}{x}\left[\psi(x-u) - \psi(x+u)\right]}$ $\qquad \int_r^\infty r\,\chi(r)\, dr = \psi(r)$

$u\, x \sin\vartheta\, d\vartheta = r\, dr$

$$\int_b^\beta = \frac{1}{\beta}\left[\psi(\beta-u) - \psi(\beta+u)\right] - \frac{1}{b}\left[\psi(b-u) - \psi(b+u)\right]$$

$\shortparallel \; 0 \qquad \shortparallel \; 0 \qquad \shortparallel \; 0$

$$dA = \frac{2\pi\rho^2\, d\sigma}{b} \underbrace{\int_0^b u\,du\, \psi(b-u)} \qquad z = b-u$$

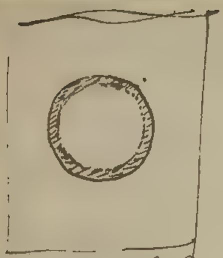
$$= b\int_0^b \psi\, dz - \int_0^b z\,\psi\, dz \qquad \psi(\infty) = 0$$

$$p_i = a\rho^2 - \frac{2\alpha\rho^2}{b} \qquad a = 2\pi\int_0^\infty \psi(z)\, dz \qquad \alpha = \pi\int_0^\infty z\,\psi(z)\, dz$$

obecně $-\alpha\rho^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ $\qquad$ vesmí poměrní

$+\alpha\rho^2\left(\frac{1}{R}\,\frac{1}{R}\right)$

dr



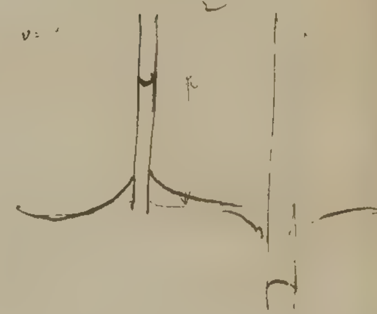
$4\pi R^2 dR$

$\Delta p = \frac{-2\alpha}{\rho g R}$

$N d\vartheta + \alpha\, dx$ ~~[illegible]~~

$\delta Q = dU + \alpha \quad dx \quad + A p\, dv$

$\cancel{[illegible]} \left[\frac{2}{R} \cdot \alpha \frac{\cancel{[illegible]}}{dx} + A p\right] \sigma\, dx$



$\Omega = 4\pi R^2$

$d\Omega = 8\pi R\, dR$

$dR = \frac{d\Omega}{8\pi R}$

$dV = \frac{R}{2}\, d\Omega$

2 sposoby parowania : w całości praca $A p\, dv$

rozdział na krople $A\left[p' + \frac{2\alpha}{R\sigma}\right] dv$

$p' = p \quad \frac{2\alpha}{R\sigma}$

$p \gtrless \frac{\alpha}{R}$

Wilhelmy

Lilienfeld (?)

$g \rho \quad \frac{z_1^2 - z_2^2}{2}$

$g \rho z \cancel{h} = \alpha\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

Trzeba

$z^2 = \text{const} - \frac{2\alpha}{\rho g} \ldots$

~~Amm...~~ $h = 75{\cdot}73 - 0{\cdot}0286\, t$

$\alpha = \alpha_0 (1 - c t)$

H_2O	$c = 0{\cdot}0019$
Alk.	$0{\cdot}0022$
H_2SO_4	$0{\cdot}0025$
[illegible]	$0{\cdot}005$

$\frac{dy}{d}$

$\frac{1}{R_1} = \infty$

$\cancel{dy} \quad \frac{ds}{d\varphi} = \frac{dy}{dy}\frac{ds}{d\varphi} =$

117

$\alpha \frac{1}{R_2} = \frac{\frac{d^2y}{dx^2}}{\sqrt{1+}^{3/2}} \alpha = y$

$y \frac{dy}{} = \alpha \, dy \, \frac{d^2y}{dx^2} =$

$\frac{y^2}{2} = \frac{\alpha}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}} = \alpha \cos\varphi + \text{const}$

$0 = \qquad \alpha \cos\varphi_0 + \text{const}$

$y^2 = \frac{2\alpha}{s}(1-\cos\varphi)$

$y = \sqrt{\frac{2\alpha}{s}}\sqrt{1-\cos\varphi}$

$= \sqrt{\alpha} \, \sqrt{}$

$Y = \sqrt{\alpha} = c$

α H_2O: 78 0°

74 20°

59 100°

Hg 436 Alk 25 Ether 19

6.1 5.4 $\frac{mm}{m}$

$2 = 15.4$

$tg\varphi = \frac{dy}{dx}$

$y = \alpha \frac{d\varphi}{dx\cos^2\varphi} \cdot \cos^3\varphi \qquad \frac{d\varphi}{dx\cos^2\varphi} = \frac{d^2y}{dx^2}$

$= \alpha \cos\varphi \frac{d\varphi}{dx} = \alpha\cos\varphi \frac{d\varphi}{dy} \, tg\varphi \qquad y^2 = a\cos\varphi + h$

$= \alpha \sin\varphi \frac{d\varphi}{dy}$

$z = \alpha \cos\varphi \frac{\partial \varphi}{\partial z} = = \alpha \frac{\frac{d^2 z}{dx^2}}{\sqrt{}^3} = \alpha \frac{d}{dx} \frac{1}{\sqrt{}} = \alpha$

z^2

$\frac{z^2}{2} = \alpha \left(h - \cos\varphi \right)$

$y_0 = \sqrt{\alpha}$

$\varphi = 0 \quad z = 0 \quad h = 1$

$\frac{z^2}{2} = \alpha (1 - \cos\varphi) = \alpha \sin \frac{\varphi}{2}$

$z = \sqrt{\alpha} \sqrt{1 - \cos\varphi}$

2

$r = r_0 + x$

$z = \frac{1}{r_0 + x} \frac{\partial}{\partial x} (r_0 \sim)$

$z = \sin \frac{\varphi}{2}$

$= \frac{1}{r}$

$\sin\varphi$

$z^2 \alpha \left(= \frac{1}{\sqrt{1 + \left(\frac{dz}{dx}\right)^2}} \right)$

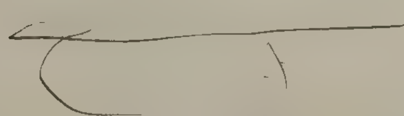
$\sqrt{1 + \left(\frac{dz}{dx}\right)^2} = \frac{1}{z^2 \alpha}$

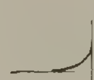
$\left(\frac{dz}{dx}\right) = \sqrt{\frac{1}{(z^2 \alpha)^2} - 1}$

Wzniesienie przy ścianie płaskiej tak samo

$h = \sqrt{2\alpha} = \sqrt{\frac{2\alpha}{\rho g}} =$ woda $\sqrt{\frac{160}{980}} = \sqrt{0.1}$

$= 0.4$ cm





3.4 mm

szerszą akcję zewnętrzną poprowadzić własnymi środkami.

Ustępując obecnie z tego miejsca cieszę się że skonstatować ~~mogę~~ trwały od wielu lat trwający ~~szybki~~ pomyślny rozwój naszego towarzystwa, ~~występujący~~ uwydatniający się ~~[illegible]~~ zwłaszcza naocznie w grafikonie Prof. Romera w ~~[illegible]~~ ciągłym przyroście liczby członków i ~~przyroście~~ ciągłym wzroście dochodów. Życzę aby rozwój w tych kierunkach i nadal trwał, aby nam z czasem umożliwił szerszą akcję zewnętrzną przez ~~podniesienie~~ krzewienie u nas ~~poziomu~~ nauk przyrodniczych i aby w równej mierze ~~wzrastała~~ się doskonaliła nasza wewnętrzna działalność naukowa, która nie zależy tak dalece od organizacji Towarzystwa, jak przedewszystkiem od naukowej kwalifikacji zapału i pojedyńczych jego członków!

Jeśli nie ma sił zewnętrznych tylko nap. pow.

~~$\Delta \frac{1}{R}$~~ $\Delta F = 0$ $\qquad p = K + \alpha\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$$\int \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \nu \, df = 0$$

błonki mydlane, równ. ciecz o jednakowej gęstości

Jeśli błonka mydl. ~~...~~ $p = 2\alpha\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

jeśli po obydwu stronach ciśn. atmosf. w komunikacji:

$$p = 2\alpha\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = 0$$

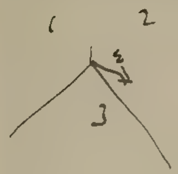
ciecz o jednak. gęstości Plateau (1843–1863) oliwa – woda + alk.

~~$\int \nu\left(\frac{1}{R_1} + \frac{1}{R_2}\right) df$~~ $\int \nu \, df = 0$ dodatkowy warunek

Ogólnie z ciężkością $\int \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \nu \, df + \nu \, df \, g \, p \left(z (\rho_1 - \rho_2)\right) = 0$

$$\int \nu \, df = 0$$

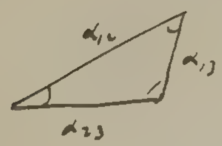
$$\frac{1}{R_1} + \frac{1}{R_2} + (\rho_1 - \rho_2) g z = 0$$

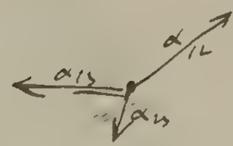


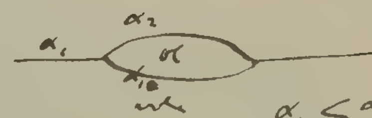
$$\int [\alpha_{12} \cos(\varepsilon n_{12}) + \alpha_{13} \cos(\varepsilon n_{13}) + \alpha_{23} \dots]\, \varepsilon\, dl = 0$$

[illegible]

[illegible] $\sin f_{12} : \sin f_{13} \dots$







$\alpha_1 < \alpha_2 + \alpha_{12}$

jeśli $\alpha_1 > \dots$

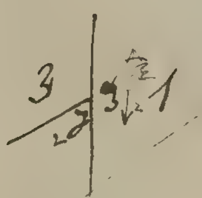
to rozpływanie

Rayleigh [illegible]

[illegible]

$\delta = 1{\cdot}6 - 0{\cdot}3$

10^{-6} cm



~~α_{12} [illegible]~~ ~~$\alpha_{12} \cos f + \alpha_{13} - \alpha_{23} = 0$~~

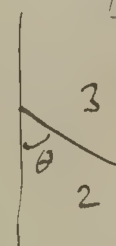
$\cos f\ \alpha_{23} + \alpha_{12} - \alpha_{13} = 0$

$$\cos f = \frac{\alpha_{13} - \alpha_{12}}{\alpha_{23}}$$

$\alpha_{13} < \alpha_{12} + \alpha_{23}$

Constance de [illegible]! [illegible]

$$\cos \theta = \frac{\alpha_{13} - \alpha_{12}}{\alpha_{23}}$$



H_2O | $8{\cdot}25$ α_1

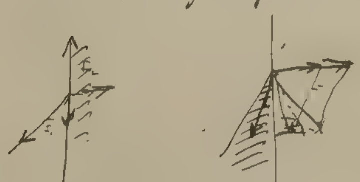
Olej $\alpha_2 = 3{\cdot}76$

$\alpha_{12} = 2{\cdot}09$

To samo z punktu napięcia powierzchni

tak samo jak wyżej.

θ [illegible] $= 138^\circ$



$F_1 \cos\theta = F_2$

$$R = \frac{ds}{d\theta} = \frac{ds}{dr}\frac{dr}{d\theta} \qquad \operatorname{tg}\theta = \frac{dz}{dr}$$

$$\frac{1}{\cos^2\theta}\frac{d\theta}{dr} = \frac{d^2z}{dr^2}$$

$$\frac{\left(1+\frac{dz}{dr}\right)^{3/2}}{\frac{d^2z}{dr^2}} = R \qquad \sin\theta = \frac{\frac{dz}{dr}}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}}$$

$$\frac{1}{R_1} = \frac{\frac{d^2z}{dr^2}}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}^{\,3}} \qquad \frac{1}{R_2} = \left(\frac{r}{\sin\theta}\right) = \frac{\frac{dz}{dr}}{r\sqrt{\cdots}}$$

$$\frac{1}{R_1}+\frac{1}{R_2} = \frac{1}{r}\frac{d}{dr}\left(\frac{r\,\frac{dz}{dr}}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}}\right)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\sin\theta) =$$

$$\frac{\sin\theta}{r} + \cos\theta\,\frac{d\theta}{dr}$$

$$\frac{\frac{d^2z}{dr^2}}{\sqrt{}}$$

$$z = \frac{a^2}{2}\,\frac{1}{r}\,\frac{\partial}{\partial r}(r\sin\theta)$$

$$r = r_0 + x$$

$$z = \frac{a^2}{2}\,\frac{\partial(\sin\theta)}{\partial x}$$

$$dz = \operatorname{tg}\theta\, dx$$

$$z^2 = -a^2\cos\theta + \text{const}$$

$$z^2 = 2a^2\left(k - \cos^2\frac{\theta}{2}\right)$$

zmienia θ jak krzywizny

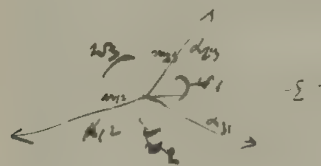
Capill.

Tropfen

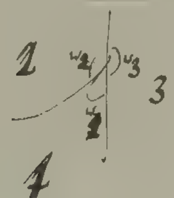
$$2\pi r\alpha = p$$

$$\int \Sigma \left[\alpha_{12} \cos(n_{12}\, \Sigma) + \alpha_{23} \cos(n_{23}\, \Sigma) + \alpha_{31} \cos(n_{31}\, \Sigma)\right] ds = 0$$

$=0$



$\sin w_1 : \sin w_2 : \sin w_3 = \alpha_{23} : \alpha_{31} : \alpha_{12}$



$n_{23} = - n_{31}$

$\cancel{\cos w_2 = \frac{\alpha_{23}\ \alpha_{31}}{\alpha_{12}}}$

Randwinkel

$\alpha_{12} \cos w_2 + \alpha_{23} - \alpha_{31} = 0$

$\cos w_2 = \frac{\alpha_{31} - \alpha_{23}}{\alpha_{12}}$

$Z = \frac{a^2}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

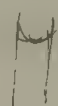
$\frac{1}{R_1} = \frac{\frac{\partial^2 z}{\partial x^2}}{\left[1 + \left(\frac{\partial z}{\partial x}\right)^2\right]^{3/2}}$

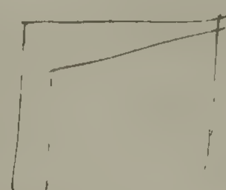
przy wielkiej krzywiźnie:

$\frac{1}{R_2} = \ldots$

dla cienkich zwykłych rurek

$z = -\frac{a^2}{R}$

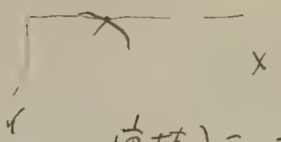




$z = -\frac{a^2}{\delta} = \ldots$

$R = \frac{r}{\cos\theta}$

2



$$\alpha = \frac{\frac{\partial z}{\partial x}}{\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}} \qquad \beta = \frac{\frac{\partial z}{\partial y}}{\sqrt{}}$$

$$\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = -\left(\frac{\partial \alpha}{\partial x}+\frac{\partial \beta}{\partial y}\right)$$

$$r = \sqrt{x^2+y^2}$$

$$\frac{\partial z}{\partial x} = \frac{x}{r}\frac{dz}{dr} \qquad \frac{\partial z}{\partial y} = \frac{y}{r}\frac{dz}{dr}$$

$$\frac{\partial}{\partial x}\left[\frac{\frac{x}{r}\frac{dz}{dr}}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}}\right] + \frac{\partial}{\partial y}\cdots$$

$$= 2\,\frac{\frac{1}{r}\frac{dz}{dr}}{\sqrt{}} + r\frac{d}{dr}\left(\frac{\frac{1}{r}\frac{dz}{dr}}{\sqrt{}}\right)$$

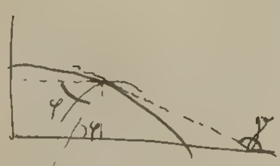
$$r\left[-\frac{1}{r^2}\frac{dz}{dr}+\frac{1}{r}\frac{d^2z}{dr^2}\right]$$

$$= \dots$$

$$= \frac{1}{r}\frac{d}{dr}\left(\frac{r\frac{dz}{dr}}{\sqrt{}}\right) = \frac{1}{r}\frac{d}{dr}\left(r\sin\psi\right)$$

$$\sin\psi = \frac{\frac{dz}{dr}}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}}$$

$$\cos\psi\frac{d\psi}{dr} = \frac{\frac{d^2z}{dr^2}}{\sqrt{}} - \frac{\left(\frac{dz}{dr}\right)^2\frac{d^2z}{dr^2}}{\sqrt{}^{\,3}} = \frac{\frac{d^2z}{dr^2}}{\sqrt{}^{\,3}}$$



$$\frac{\frac{d^2z}{dr^2}}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}^{\,3}} + \frac{\sin\psi}{r}$$

$$R_2 = \frac{r}{\cos\varphi} = \frac{r}{\sin\psi}$$

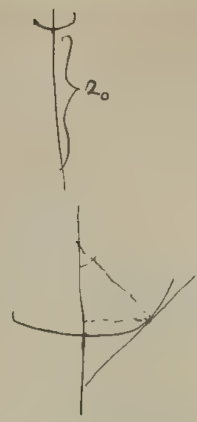
$$\varphi + (\pi-\psi) = \frac{\pi}{2} \qquad \varphi = \psi - \frac{\pi}{2}$$

$$\cos\psi\frac{d\psi}{dr} \qquad \cos\psi\frac{d\psi}{dr}+\frac{\sin\psi}{r}$$

$$\frac{d\psi}{ds} = \frac{d\psi}{\frac{dr}{\cos\psi}}$$

$$= \frac{x}{r}\frac{d}{dr}\left(r\sin\psi\right)$$

u bardzo duże, 2 duże $\neq z_0$



$$z_0 r = \alpha^2 \frac{d\, r \sin\theta}{dr}$$

$$z_0 r + \frac{c}{r} = \alpha^2 \sin\varphi$$

$$z_0 r = \alpha^2 \sin\varphi$$

kula

promień $= \frac{\alpha^2}{z_0}$

$$\frac{1}{r}\int_0^r z\, r\, dr = \alpha^2 \sin\theta = \frac{1}{r}\int_0^r (z_0+\zeta)\, r\, dr$$

$$= \frac{r}{2} z_0 + \frac{1}{r}\int_0^r \zeta\, r\, dr$$

$$r\zeta = R(1-\cos\varphi)\, r = \frac{\alpha^2}{z_0}(1-\cos\varphi)\frac{\alpha^2 \sin\varphi}{z_0}$$

$$\frac{1}{\sin\theta}\int_0^{\varphi}(1-\cos\theta)\sin\theta\cos\theta\, d\theta$$

$$z_0 r = 2\alpha^2 \sin\varphi - \left(\frac{2\alpha^2}{z_0}\right)^2\left[\sin\varphi - \frac{2}{3}\frac{1-\cos^3\varphi}{\sin\varphi}\right]$$

~~taki~~ tak ~~siła~~ ciężar

$$\alpha\, 2\pi R \sin\varphi = R^2\pi\rho g z - \frac{2}{3}R^3\pi$$

$$z = \frac{2\alpha\cos\varphi}{\rho R}$$

woda

$$z_0 R = 2\alpha - \frac{1}{3}\left(\frac{2\alpha}{z_0}\right)^2$$

$$+ 2\alpha - \frac{R^2}{3}$$

$$z_0 \neq \frac{2\alpha}{R} - \frac{R}{3}$$

H_2O $\alpha =$	79
Hg	480
.	26
alk.	20
eter	33
oliwa	33
CS_2	

$\psi = \frac{R\theta}{p}$

$$\delta\left(4\pi x^2 \alpha + \frac{e^2}{2r}\right) + \int 4\pi r^2 dr \int_p^P p\, dv = 0$$

$$\underbrace{}_{R\theta \lg \frac{P}{p}}$$

$$R\theta \lg \frac{P}{p} = \frac{1}{4\pi r^2} \frac{d}{dr}\left(4\pi r^2 \alpha + \frac{e^2}{2r}\right)$$

$$= \frac{2\alpha}{r} - \frac{e^2}{8\pi r^4} \qquad c = \sqrt[3]{\frac{e^2}{16\pi\alpha}}$$

$$= 2\alpha\left(\frac{1}{r} - \frac{c^3}{r^4}\right) = \frac{2\alpha}{c}\left(\frac{c}{r} - \frac{c^4}{r^4}\right)$$

$$= \frac{2\alpha}{c}(x - x^4)$$

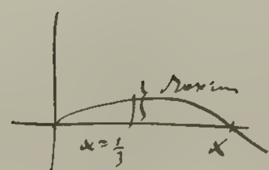
$H_2O \quad \alpha = 76$

$e =$

$n = \frac{1}{3 \cdot 2} 10^{-7}$

$e = 3 \cdot 10^{-10}$

$$\sqrt[3]{\frac{9 \cdot 10^{-20}}{16 \cdot 3 \cdot 80}} = \cancel{\sqrt[3]{\frac{10^{-20} \cdot 3}{8 \cdot 16}} = \sqrt[3]{\frac{10^{-21}}{400}}} =$$



$$R\theta \lg \frac{P}{p} = \frac{2\alpha}{c}\, 0.471$$

$$\frac{p}{P} = 5.3$$

$\frac{v_2}{v_1}$ 1.25

1.31

1.38

CTR Wilson 4-5

$$\cancel{T = \frac{p}{\rho} = \rho^{k-1}}$$

$$\frac{T}{T} = \left(\frac{v}{v}\right)^{k-1} = (1.25)^{0.4}$$

$$= 1.1 = \cancel{290 \times 2}.$$

$20° - 1 - 10°$

Sidrowsky $\bigcirc \ p = \frac{2\alpha}{R}$ $\alpha = 80$
dla $R = 10^{-}$

~~[illegible]~~ [illegible] $p = P - \frac{\alpha}{R} \frac{\rho_L}{\rho_w}$

Rayleigh Elektrodynamics [illegible] Kanfoly

0·81 my $\left(\frac{8\alpha}{4}\right)^2 \pi / \mu m^2$ $\delta = 1{\cdot}6.\ 10^{-8}$ cm

δ

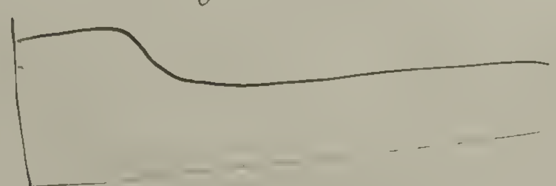
0·40 my	$\delta = 0{\cdot}81$	no effect
0·52	1·06	barely perceptible
0·65	1·32	not quite enough
0·78	1·58	just enough

Hg α = 55·03
H_2O 8·25

α_{12} = 42·58

Rayleigh weight of drops IV p. 420

Pockels Rayleigh -



~~woda (Krapp)~~ Tt.

$\alpha = 79\,(1 - 0{\cdot}002\,t)$

H_2O		Alk.		Ath.	
0°	79	0°	26	0°	20
100°	62	75°	19	35°	16

woda wielka od innych gazów

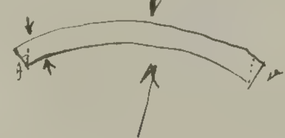
$$\boxed{CO_2 \quad h = 26{\cdot}04 - 0{\cdot}825\,t}$$

$$= 0 \qquad t = 31{\cdot}5$$

$\alpha\,(Mv)^{2/3}$ molekulare Oberfl. Energie   Eötvös Ramsay & Shields

$$= k(\vartheta - t - d) \qquad k = 2$$

$(d = 6)$



$$\int\left(\frac{1}{R_1} + \frac{1}{R_2}\right) dx\, dy$$

$$= \left[\left(\frac{1}{R_1} + \frac{1}{R_2}\right) df \cos\ldots\right] = \frac{\Delta F}{v}\cos(n z) = \frac{dl \cdot v \cdot \cos(nz)}{v}$$

$\cos nz = 1$

$$2\,dx = \int l\,\alpha \qquad = dl$$

Odwrotnie pływanie igieł, pająków

Roztwory Quincke wszystkie $\alpha = 77 + 0{\cdot}16\,y$

spróbujmy wpływ CCl_4, Äther etc. oleju   zapewne połączenia się warstwy powierzchniowej

Kamfora na wodzie

Kołowski ~~emulsje~~ roztwory koloid.

Range of molec. Forces Quincke

$l > 0{\cdot}000054$ mm H_2O/Ag/ etc.

$$z = a\,\frac{\frac{d^2z}{dx^2}}{\sqrt{1+\left(\frac{dz}{dx}\right)^2}^{\,3}} = a\,\frac{d}{dx}(\sin\varphi)$$

$\int z\,dx = a \sin\varphi \Big|^{2}$ dla cieczy zwilżającej : $= a = \frac{\alpha}{\rho g}$

waga ciężar $\rho g \int z\,dx = \alpha$ oczywiście ! ~~równ~~ równowaga sił

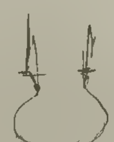
metoda Wilhelmy : płytki platyny wyciąganie ciężarem ponowne

Błędy areometrów

Adhäsionsplatten :

tutaj opiera napięcie pow. na brzegu jeżeli ciężar hydrostat. na powierzchnię

Krople (Quincke)

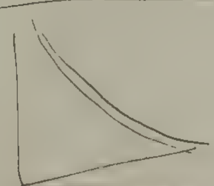


$p = 2\pi r \alpha$ podobnie jak przy metodzie baniek ?

nie dokładnie bo nie równy się ciężar kropli oderwanej wszystkiej ~~wszystkiej~~ równa się ciężarowi kropli odrywającej

znaczny błąd jeśli krople nie szybko tworzą się tylko przy powolnym.

dokładniej na trzecią α obliczeniu — —



$$z = \left(\frac{1}{R_1}+\frac{1}{R_2}\right)\frac{\alpha}{\rho g} = \frac{2}{r}\,\frac{\alpha}{\rho g} = \frac{2c}{x}\,\frac{\alpha}{\rho g}$$

płytki równoległe !

wysokość $= \frac{1}{2}$ rurki

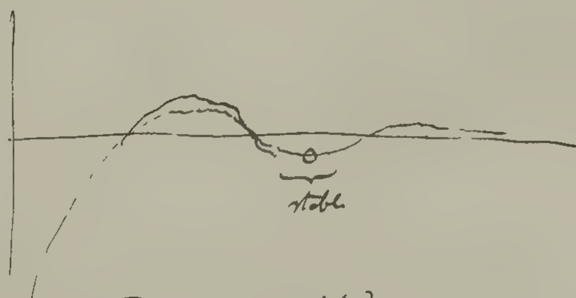
siła ściskająca je

[illegible]

$$= \frac{2\alpha}{r} + \frac{\partial\sigma}{\partial r} - \frac{e^2}{8\pi r^4}$$

Reinold x Rücker

Ostbiene Alkohol



$$\left(\frac{e}{r^2} + \frac{V}{d}\right)^2$$

[illegible]

[illegible]

1) [illegible] 1741 [illegible] – 1786

1787 – [illegible] + 1813

[illegible] 1789 [illegible]

[illegible] 1804

[illegible]

2) [illegible] 1750 [illegible]

– 1790

[illegible] 1781 – 1784 + 1826

3). Anton [illegible]

4). [illegible] – 1806

[illegible]

[illegible]

[illegible]

7. [illegible] 1813 [illegible]

[illegible]

8. [illegible]

9. [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] 1813 [illegible]

[illegible]

1809 [illegible]

Unruhen 1811 [illegible]

1813 promoving der Kol physic. [illegible] 1818 [illegible] Tobarki

ZAKŁAD FIZYCZNY
C. K. UNIWERSYTETU JAGIELLOŃSKIEGO

Kraków
Gołębia 13.

S. Hirzel Leipzig

Physikalische Zeitschrift

Gewünscht wird:

5. Jahrgang Heft № 16

~~8. " " 9~~

14. 5

~~13. " " 20~~

Abzutreten sind Duplikate:

I	Jahrg. Heft №	1 u. 2.
2		20
4		1, 26 b
6		17
13		18, 21/22

1). Loria
Disp. d. ... Bull. 1904

2). ... Z. f. ...

3). Zbadanie zagadnień i metod psychologji · Poradnik samouka 1909

4). Die Disp. ... CO_2 (Z. S. Zaodal.) Verh. D. ph. G. X (1908) Wiad. mat. 13 (1909)

5). ... Disp. ... I Bull. 1909

6). " II 1909 Ann. d. Ph. 29 (1909)

7). Die Disp. ... Na ... Bull. 1909 Ann. 30 (1909)

8). O dysp. światła w parach metali R A U (1910)

9). The magneto optic Kerr effect in ferro magn. comp. and alloys I { O. Amst. 1910 / Kosmos 35 (1910)

10). Die opt. Konstanten ~~...~~ ... magn. opt. Kerr-Phän. ... (Z. S. Zakrz.) Bull. 1910

11). The magnet. optic Kerr Eff. in ferromagn. compounds and alloys II O. A. 1912
Ann. d. Ph. 38 (1912)

12). ... Disp. ... III (Z. S. Patk.) Bull. 1913

13). O rozpraszaniu i dysp. światła w gazach Prace mat. fiz. 25 (1914) w druku

$C_7H_6O_2 + KClO_3 =$

Galton Fr FRS Memories of my Life 10/6 Methuen

Mill J St. Autobiography 3/6 popular ed. -/6 Longman & Co

William Morris by Alfred Noyes 2/- Macmillan

As others see us, A study of Progress in the U.S. J. G. Brooks 7/6 "

The Fijians: a study of the Decay of Custom B. Thomson 10/- Heinemann

Essays Biographical & Chemical Sir W. Ramsay 7/6 } A. Constable

First & last things A confession of faith & rule of life H.G. Wells 4/6 }

Morley: Life of Gladstone 2 vols 5s. with post 5/6 Daily Chronicle 12 Salisbury Sq. E.C.

~~wszystkie do tego potrzebne wiadomości i chodzi tylko o ich uogólnienie i usystemizo-wanie, a korzyść jest bezporównania większa niż n.p. z przerobienia~~

Phys Z [13] D. Physik. Inst. d. Univ. Leipzig

Praktikantenwerkstätte

Projektion vom Tische d. Hörsaals auf eine Gipsfläche auf der Wand hinter

Experimentiertisch 5 × 5 m ; Von derselben Tafel, welche weggeschoben werden kann

El. Zuleitungen 30 – 50 Amp. || Pressluft | Dampfltg | 1200 Amp.
im ganzen

Hochspannungs Dynamo
10.000 V. 10' Amp.

Brett auf (Kugellagern / ~~Schienen~~, Rädern) zur Demonstr. d. Sätze d. Schwerp.
bei Fortschreiten ... rollen

Sammlg 8 Schränke 4 × 1.50 × 2.70

Bei Neubau bewilligt 102.070 Mk. zur Anschaffung neuer Apparate & Möbel

schliesslich jedoch wurde flüssig gemacht : 160.300 Mk App.)) !
10 000 Mk Bibl.))

900 Bde. Zeitschr.
26 deutsche Zeitschr.
9 ausländische
jährl. ca 2000 M

Werkstätte : 5 Drehbänke (3 elektr.)

Praktikantenwerkstätte : Drehbank, Hobelbank, Schleifstein etc.

Pressluft 10 Atm.
Saugluft 10 m.

24 Telephonstationen 18 Klingeln
Strom gleichzeitig überall
auf Norm-Anruf

Kosten : Bauareal 79.000
Baukosten 858.000
einschl. Heizung, Gas, Wasserltg

Innere Ausstattung Mobiliar 81.000
Elektr. Anlage 79.—
Bibl. 10.—
Apparate 160.—
Apparate f. Übungen 96.—
426.000

1.363.000
Ersparnis gegenüber Voranschlag
69.000

Kolloq. 2 St. wö (aufeinanderfolgend) Wechsel d. beiden Direktoren

Dotation für Apparate, Instrumente, Bücher, Verbrauchsgegenstände : 18.000
+ 1400 (Praktikantentaxen)

Heizung, Wasser, Gas, Elektr. u. Reinigung : 15.000

5 Assistenten + 2 Hilfsassistenten

Vorlesungsbesuch 338 Praktikanten 188 Selbständige Arbeitende
18 + $\underbrace{10}_{\text{Doktoranden}}$

Hörerzahl d. Universität 5008

Hilfspersonal 6 + Hilfsheizer (½ jährig) ; Privatdozent Dr. Scholl, Dr. [illegible] etc.

+ (3) aus Pauschal der Dotation bezahlt
+ [3] ohne Gehalt (Lehrlinge)

Direktor: Dienstwohnung

Th Ph I 96.000 f Instit. Assist. u Werkstatt

1 Assistent
1 Mechaniker u
3 Lehrlinge nicht bezahlt

Jahresdotation 4.000 [Pauschal genommen .. ist ...]

(5 - 7) → Praktikanten kaum 15 Std. pro Sem..
selbständig

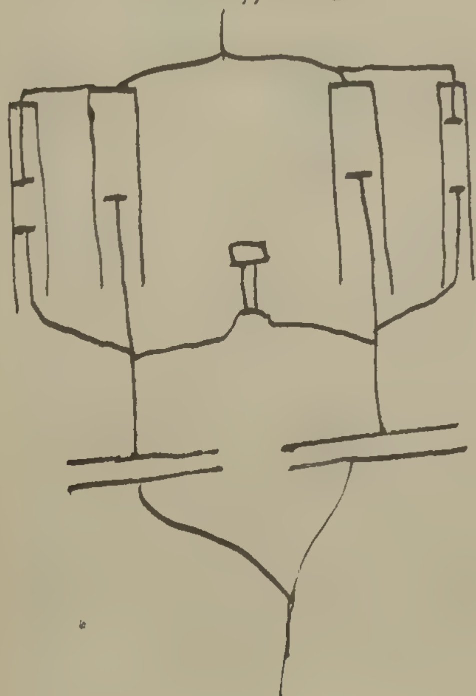
dazu: 70 Hörer d. kurzen Kurses Exp. 4 St.

25 speziellen 2 St.

$$C + C_2 = C_{11}$$

$$C_2 = C_{12} + C.$$

$$2C = C_{11} - C_{12}$$



$$\cancel{a_2^2\mu_2 = (v_1x - u_1y_1)} - a_1^2\mu_1 + (\mu_2-\mu_1)\frac{(c^2-a_1^2-a_2)}{2}$$

$$\mu_2 a_2^2 = \frac{v_1x_1 - u_1y_1 - a_1^2\mu_1 + (\mu_2-\mu_1)\dfrac{c^2-a^2}{2}}{\mu_2 + \dfrac{\mu_2-\mu_1}{2}}\mu_2$$

$$a_2^2\mu_2 = \frac{U}{\mu_1+\mu_2} - a_1^2\mu_1$$

$$(v_1x_1 - u_1y_1)\mu_2 - a_1^2\mu_1\mu_2 + (\mu_2-\mu_1)\mu_2\frac{c_1^2-a_1^2}{2} = \frac{3\mu_2-\mu_1}{2}\left[\frac{U}{\mu_1+\mu_2} - a_1^2\mu_1\right]$$

$$\left[v_1x_1 - u_1y_1 + \frac{(\mu_2-\mu_1)c^2}{2}\right]\mu_2 - \frac{3\mu_2-\mu_1}{2}\frac{U}{\mu_1+\mu_2} = a_1^2\left[\mu_1\mu_2 + (\mu_2-\mu_1)\frac{\mu_2}{2} - \frac{3\mu_2-\mu_1}{2}\mu_1\right]$$

$$\frac{2\mu_1\mu_2 + \mu_2^2 - \mu_1\mu_2 - 3\mu_1\mu_2 + \mu_1^2}{2}$$

$$\frac{\mu_1^2 + \mu_2^2 - 2\mu_1\mu_2}{2}$$

$$= a^2\frac{(\mu_1-\mu_2)^2}{2}$$

$$a_1^2\mu_1 - a_2^2\mu_2 = \frac{-U}{\mu_1+\mu_2} + \frac{4\mu_1}{(\mu_1-\mu_2)^2}\left\{-\frac{3\mu_2-\mu_1}{2}\frac{U}{\mu_1+\mu_2} + \mu_2\left[v_1x_1 - u_1y_1 + \frac{\mu_2-\mu_1}{2}c_1^2\right]\right\}$$

$$= \frac{-U}{\mu_1+\mu_2}\left\{1 - \frac{2\mu_1(3\mu_2-\mu_1)}{(\mu_1-\mu_2)^2}\right\}$$

$x = f_0(t, R_0, a_0, \alpha_0 \ldots)$

$x = f_{R_0=0}(t) + R_0 \left(\frac{\partial f}{\partial R}\right)_{R_0=0} \ldots$

Ale ~~[illegible]~~ (jeśli $\frac{\partial x}{\partial t}$ nieciągłe dla $t=0$, co tutaj ma miejsce

jeśli pole R powstaje natychmiastowo w momencie $t=0$.

$f_{R_0=0}(t)$ nie będzie równa $f\binom{t=0}{R_0=0}$)

[illegible] stacyon II · I [illegible] [illegible]

za czas $t=0$:

$$x_0 = a\cos\varepsilon = x_1 \qquad u_1 = -a\alpha\sin\varepsilon + \frac{eR\tau}{2m\omega}\,a\alpha\cos\varepsilon$$

$$y_0 = a\sin\varepsilon = y_1 \qquad v_1 = a\alpha\cos\varepsilon + \frac{eR\tau}{2m\omega}(\omega + a\alpha\sin\varepsilon)$$

co nam [illegible] [illegible] z [illegible] [illegible] (27):

$$x_1 = a_1\cos\alpha_1 + a_2\cos\alpha_2 = a\cos\varepsilon$$

$$y_1 = a_1\sin\alpha_1 - a_2\sin\alpha_2 = a\sin\varepsilon$$

z których wyznaczamy $a_1\, a_2\, \alpha_1\, \alpha_2$ ograniczając się na [illegible] [illegible] [illegible]

$$u_1 = -a_1\mu_1\sin\alpha_1 - a_2\mu_2\sin\alpha_2 = -a\alpha\sin\varepsilon + \frac{eR\tau}{2m\omega}\,a\alpha\cos\varepsilon$$

$$v_1 = a_1\mu_1\cos\alpha_1 - a_2\mu_2\cos\alpha_2 = a\alpha\cos\varepsilon + \frac{eR\tau}{2m\omega}(\omega + a\alpha\sin\varepsilon)$$

$$\mu_1 = \mu\left[1 + \frac{}{} \right] = \mu\left[1 \mp \frac{1}{2}\frac{\pi\,\ldots}{\mu\ldots}\right] = \alpha\left(1 - \frac{eR}{2m\omega\alpha}\right)$$

$$\mu_2 = \qquad = \alpha\left(1 + \frac{eR}{2m\omega\alpha}\right)$$

$$-a_1\alpha\sin\alpha_1 \cancel{- a_2\alpha\sin\alpha_2} + \frac{a_1 eR}{2m\omega\alpha}\sin\alpha_1 - \frac{a_2 eR}{2m\omega\alpha}\sin\alpha_2 = -a\alpha\sin\varepsilon + \frac{eR\tau}{2m\omega}a\alpha\cos\varepsilon$$

Moment [illegible] (wzór 30): $\mu_1 a_1^2 - \mu_2 a_2^2 = \alpha\left[(a_1^2 - a_2^2) - \frac{eR}{2m\omega\alpha}(a_1^2 + a_2^2)\right]$

Moment początkowy : αa^2

[illegible]

$$m\frac{d^2y}{dt^2} + \dots = \frac{e\lambda}{\dots}\left(-\frac{dx}{dt} + \dots\right)$$

$$y = a_1 \sin(p_1 t + \alpha_1) - a_2 \sin(p_2 t + \alpha_2) + \frac{e\lambda}{k\omega}$$

[illegible]

$$x_1 = a_1 \cos\alpha_1 + a_2 \cos\alpha_2$$

$$y_1 = a_1 \sin\alpha_1 - a_2 \sin\alpha_2 + \frac{e\lambda}{k\omega}$$

$$-u_1 = a_1 p_1 \sin\alpha_1 + a_2 p_2 \sin\alpha_2 \qquad u_1^2 + v_1^2 = a_1^2 p_1^2 + a_2^2 p_2^2 + 2a_1 a_2 p_1 p_2 (\sin\alpha_1 \sin\alpha_2 - \cos\alpha_1\cos\alpha_2)$$

$$v_1 = a_1 p_1 \cos\alpha_1 - a_2 p_2 \cos\alpha_2 \qquad V_1^2 =$$

$$c_1^2 = x_1^2 + y_1^2 = a_1^2 + a_2^2 + 2a_1 a_2 (\cos\alpha_1 \cos\alpha_2 - \sin\alpha_1 \sin\alpha_2)$$

$$V_1^2 + c_1^2 p_1 p_2 = a_1^2 p_1^2 + a_2^2 p_2^2 + a_1^2 p_1 p_2 + a_2^2 p_1 p_2$$

$$= \cancel{a_1^2 p_1 (p_1 + p_2) + a_2^2 p_2 (p_1 + p_2)} = (a_1^2 p_1 + a_2^2 p_2)(p_1 + p_2)$$

$$v_1 x_1 = a_1^2 p_1 \cos^2\alpha_1 - a_2^2 p_2 \cos^2\alpha_2 + a_1 a_2 \cos\alpha_1 \cos\alpha_2 (p_1 - p_2)$$

$$u_1 y_1 = -a_1^2 p_1 \sin^2\alpha_1 + a_2^2 p_2 \sin^2\alpha_2 + a_1 a_2 \sin\alpha_1 \sin\alpha_2 (p_1 - p_2)$$

$$v_1 x_1 - u_1 y_1 = \cancel{a_1^2 p_1 - a_2^2 p_2}$$

$$= a_1^2 p_1 + a_2^2 p_2 + a_1 a_2 (p_1 - p_2)(\cos\alpha_1 \cos\alpha_2 - \sin\alpha_1 \sin\alpha_2)$$

$$= a_1^2 p_1 + a_2^2 p_2 + (p_1 - p_2)\frac{(c_1^2 - a_1^2 - a_2^2)}{2} = \cancel{a_1^2 p_1}$$

[illegible] :

$$m \frac{d^2x}{dt^2} + k x = [illegible]$$

$$m \frac{d^2y}{dt^2} + k y = [illegible]$$

[illegible]

[illegible]

$$[illegible] = -\frac{e R}{4m^2\omega [illegible]} \left(m\, [illegible] - k\, [illegible] \right) = -\frac{e R}{4m^2\omega \frac{k}{m}} \left[m(u_0^2+v_0^2) - k(x_0^2+y_0^2) \right]$$

$$= \frac{eR}{4m\omega} \left[x_0^2 + y_0^2 - \frac{m}{k}(u_0^2+v_0^2) \right]$$

$v_1 = v_0 + \Pi\tau(\omega - u_0)$ $\quad x_1 = x_0$

$u_1 = u_0 + \Pi\tau\, v_0$ $\quad y_1 = y_0$

$$(v_1 x_1 - u_1 y_1) - (v_0 x_0 - u_0 y_0) = \Pi\tau(\omega x_0 - u_0 x_0 + v_0 y_0)$$

$$x = a_1 \cos(pt + \alpha_1) - a_2 \cos(pt + \alpha_2) + \Pi\left[a_1 \sin(pt + \alpha_1) - a_2 \sin(pt + \alpha_2)\right]$$

$$y = a_1 \sin(pt + \alpha_1) + a_2 \sin(pt + \alpha_2) + \Pi\left[-a_1 \cos(pt + \alpha_1) - a_2 \cos(pt + \alpha_2)\right]$$

$$u = -p a_1 \sin(pt + \alpha_1) - a_2 p \sin(pt + \alpha_2) + \Pi p\left[a_1 \cos(pt + \alpha_1) - a_2 \cos(pt + \alpha_2)\right]$$

$$v = p a_1 \cos(pt + \alpha_1) + a_2 p \cos(pt + \alpha_2) + \Pi p\left[a_1 \sin(pt + \alpha_1) + a_2 \sin(pt + \alpha_2)\right]$$

Resistance of Ellipsoid:

$$R = 16\pi\mu u \frac{1}{\int_0^\infty \frac{d\lambda}{\Delta} + a^2 \int_0^\infty \frac{d\lambda}{(a^2+\lambda)\Delta}} \qquad \Delta = (a^2+\lambda)\sqrt{c^2+\lambda}$$

$$\frac{1}{\alpha - c^3}\left[\frac{2\alpha - c^3}{\sqrt{\frac{\alpha}{c} - c^2}} \left[\frac{\pi}{2} - \arccos\sqrt{1-\left(\frac{c}{a}\right)^2}\right] - \frac{c^4}{a^2}\right]$$

$$\frac{1}{a^2-c^2}\left[\frac{2a^2-c^2}{\sqrt{a^2-c^2}} \arccos\sqrt{1-\left(\frac{c}{a}\right)^2} - \frac{c^3}{a^2}\right] \quad \Bigg| \quad \frac{1}{c^2-a^2}\left[\frac{c^2-2a^2}{2\sqrt{c^2-a^2}} \lg \frac{c+\sqrt{c^2 - \dots}}{c-\sqrt{}} + \frac{c^3}{a^2}\right]$$

$$ac^2 = \alpha \qquad \frac{c}{a} = 1-\delta$$

$$c^3 = \alpha(1-\delta)$$

$$c = \sqrt[3]{\alpha}\,(1-\delta)^{1/3}$$

$$a = \sqrt[3]{\alpha}\,(1-\delta)^{-2/3}$$

$$\frac{1}{a}\left\{-\frac{1}{1-(1-\delta)^2} + \frac{2-(1-\delta)^2}{\left[1-(1-\delta)^2\right]^{3/2}} \lg \frac{1+\sqrt{2\delta \dots}}{1-\sqrt{\dots}}\right\}$$

$$\lg \frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5}\right)$$

$$-1 + (1+2\delta-\delta^2)\left(1 + \frac{2\delta-\delta^2}{3} + \frac{(2\delta-\delta^2)^2}{5}\right) \qquad N = \frac{\left(1-\frac{2\delta}{3}\right)}{1-\frac{\delta}{2}}\left(1+\frac{3}{10}\delta\right)$$

$$-\cancel{1} + \cancel{1} + 2\delta - \cancel{\delta^2} + \frac{2\delta}{3} + \frac{4\delta^2}{3} - \frac{\delta^2}{3} + \frac{4\delta^2}{5} \qquad 1 - \frac{2}{3}\delta + \frac{\delta}{2} + \frac{3}{10}\delta$$

$$\frac{8\delta}{3} + \cancel{\frac{-15+20-5}{15}} + \frac{4\delta^2}{5} \qquad \frac{-20+15+9}{30}$$

$$\frac{c}{a} = 1+\delta \qquad a = \sqrt[3]{\alpha}\,(1+\delta)^{-2/3}$$

$$N = \frac{(1+\delta)^{2/3}}{\sqrt[3]{\alpha}\,(2\delta+\delta^2)}\left[1 + \frac{(1+\delta)^2 - 2}{(2\delta+\delta^2)^{1/2}}\left[\underbrace{\frac{\pi}{2} - \arccos\sqrt{1 - \frac{1}{(1+\delta)^2}}}_{\arcsin\sqrt{1-\frac{1}{(1+\delta)^2}}}\right]\right]$$

$$1 + (\underline{2\delta+\delta^2-1})\left(\frac{1}{1+\delta} + \frac{2\delta+\delta^2}{3\,(1+\delta)^3} + \frac{3}{2\cdot4\cdot5}\,4\delta^2\right) \qquad \frac{\sqrt{2\delta+\delta^2}}{1+\delta}$$

$$1-\delta+\delta^2 + \frac{2\delta+\delta^2}{6}(1-3\delta) + \frac{3}{10}\delta^2$$

$$1-\delta+\cancel{\delta^2}+\frac{\delta}{3}+\frac{\delta^2}{6}-\cancel{\delta^2}+\frac{3\delta^2}{10} \qquad \frac{+5+9}{30} \quad \frac{14}{30}$$

$$1-\frac{2\delta}{3}+\frac{7\delta^2}{15}$$

$$\cancel{1}-\cancel{1}+\frac{2\delta}{3}-\frac{7\delta^2}{15}+2\delta-\frac{4\delta^2}{3}+\delta^2$$

$$\frac{8\delta}{3}+\frac{15-20-7}{15} = \frac{8\delta}{3}-\frac{4}{5}\delta^2 = \frac{8}{3}\delta\left(1-\frac{49}{80}\delta^2\right)$$

$$\left(1+\frac{2\delta}{3}\right)\left(1-\frac{\delta}{2}\right)\left(1-\frac{49}{80}\right) \qquad \frac{160-120-147}{240} \qquad \frac{-107}{240}$$

$$\left(1+\frac{2\delta}{3}\right)\left(1-\frac{\delta}{2}\right)\left(1-\frac{3}{10}\right)$$

$$\frac{+20-15-9}{30}$$

Resistance of Ellipsoid:

$$F = 16\pi\mu U \frac{1}{\displaystyle\int_0^\infty \frac{d\lambda}{\Delta} + a^2 \int_0^\infty \frac{d\lambda}{(a^2+\lambda)\Delta}} \qquad \Delta = \sqrt{(a^2+\lambda)(b^2+\lambda)(c^2+\lambda)}$$

for ~~rotation~~ axial symmetry $b = c$

$$F = 16\pi\mu U \frac{1}{\displaystyle\int_0^\infty \frac{d\lambda}{(c^2+\lambda)\sqrt{a^2+\lambda}} + a^2 \int_0^\infty \frac{d\lambda}{(c^2+\lambda)(a^2+\lambda)\sqrt{a^2+\lambda}}} = \frac{16\pi\mu U}{N}$$

By putting $\cancel{a^2+\lambda = x^2}$

$$\cancel{d\lambda = 2x\,dx} :$$

$$\cancel{N = 2\int_a^\infty \frac{dx}{x^2+c^2-a^2} + a^2 \int_a^\infty \frac{dx}{x^2(x^2+c^2-a^2)}} = \left(\frac{1}{x^2} - \frac{1}{x^2+c^2-a^2}\right)\frac{1}{c^2-a^2}$$

$$\frac{\partial}{\partial a} \int_0^\infty \frac{d\lambda}{(c^2+\lambda)\sqrt{a^2+\lambda}} = -\int_0^\infty \frac{a\,d\lambda}{(c^2+\lambda)\sqrt{a^2+\lambda}^3}$$

$$N = \left[1 - a\frac{\partial}{\partial a}\right] \int_0^\infty \frac{d\lambda}{(c^2+\lambda)\sqrt{a^2+\lambda}} = 2\left[1 - a\frac{\partial}{\partial a}\right] \int_a^\infty \frac{dx}{x^2+c^2-a^2}$$

I). $c < a$

$$\int_a^\infty \frac{dx}{x^2-(a^2-c^2)} = \frac{1}{\sqrt{a^2-c^2}} \log \frac{a+\sqrt{a^2-c^2}}{a-\sqrt{a^2-c^2}}$$

$$\frac{\partial}{\partial a} \; - - - - = \frac{-a}{\sqrt{a^2-c^2}^3} \log \frac{a+\sqrt{\ }}{a-\sqrt{\ }} + \frac{1}{\sqrt{a^2-c^2}}$$

86·67 0·5547
508 ·7059
35·87 0·8488

620 8·6 2
0·7060

870 2·5
703

2·17 || 7032
7060

0·5
3

3365 6730
1644 9138
5009 5868

2·8
50·8 0
62·8 1·0
64·0 1·2 1·1

1·2
13·2

3171
3861
7032

0·198 09
0·2047

0170
7059
3111
15

0·146
82
0·228 1·0

0·260 1·1

090

8542 9084
1644 9138
1186 8222

1314
664
0·198

6·7
30 0·1

0·922 0·22

94·87
508
44·07

6442
7059
9383

0·8685
0·878 — 2·5
0823 2·4

3802 7604
1644 9138
5446 6742

10
55 0·248

3504
4723
8227

2041 4082
1644 9138
3685 3220

2336
2099
4435 1·6

74·7
508
23·9

3784
7059
6725

4704

404 1·5

2304 4608
1644 9138
3948 3746

485 1·7
15
4704

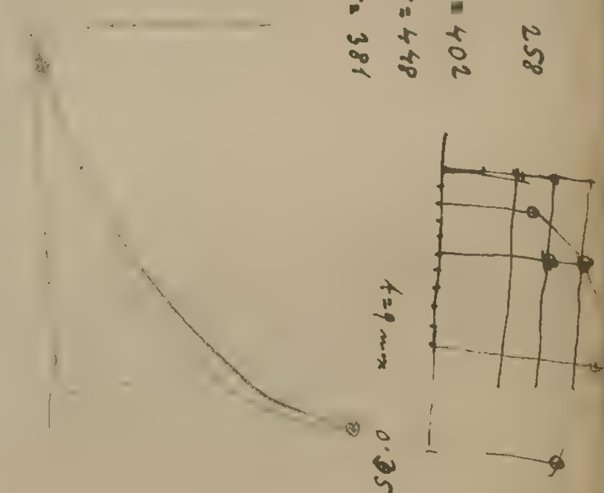
4435 1·6

2482
2369
485

1·67

146
415 = 0·3

1905 : 5 = 381
2242 : 5 = 448
1207 : 3 = 402
517/2 = 258



$$\frac{\partial p}{\partial x} = \frac{\mu}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \frac{p_1 - p_2}{l}$$

$$u = \frac{p_1 - p_2}{4\mu l}(R^2 - r^2)$$

$$-J = \frac{R^4\pi}{8\mu}\ \frac{p_1 - p_2}{l}$$

$$\cancel{u = \frac{1}{\mu}\left\{-\frac{r^2}{6}\frac{\partial}{\partial x}\left(\frac{A x}{r^3}\right) - \frac{2}{3} r \frac{\partial}{\partial x}\right.}$$

$$u = -\frac{3}{4}\frac{ca}{r^3}\left(1 - \frac{a^2}{r^2}\right)x^2 + c\left(1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right)$$

$$v = -\frac{3}{4}\frac{ca}{r^3}\left(1 - \frac{a^2}{r^2}\right)xy$$

$$w = -\frac{3}{4}\frac{ca}{r^3}\left(1 - \frac{a^2}{r^2}\right)xz$$

$$p = p_0 - \frac{3}{2}\mu\frac{ca\,x}{r^3}$$

$$\Delta^2[\varphi(r)\cdot x] = x^2\Delta^2\varphi + 4x\frac{\partial\varphi}{\partial x} + 2\varphi$$

$$\Delta^2[\varphi\cdot xy] =$$

$$\Delta^2\left(\frac{xy}{r^3}\right) \qquad \Delta^2\left(\frac{x^2}{r^3}\right) \qquad \Delta^2\left(\frac{1}{r^3}\right)$$

$$\left(\frac{xy}{r^5}\right) \qquad \frac{x^2}{r^5}$$

$$\cancel{\frac{\partial}{\partial x}\left(\frac{x^{m+1}y}{r^3}\right) = \frac{y}{r^3}} \qquad \frac{\partial}{\partial x}\left(\frac{x^m y^n}{r^p}\right) = m\frac{x^{m-1}y^n}{r^p} - p\frac{x^{m+1}y^n}{r^{p+2}} \qquad p\frac{z}{r^{p+2}}\ x^m y^n$$

$$\frac{\partial^2}{\partial x^2} = m(m-1)\frac{x^{m-2}y^n}{r^p} - [p(m+1) + pm]\frac{x^m y^n}{r^{p+2}} + p(p+2)\frac{x^{m+2}y^n}{r^{p+4}}$$

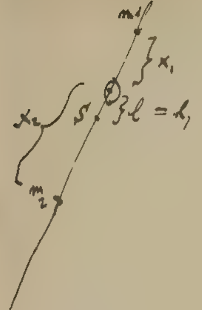
$$n(n-1)\frac{x^m y^{n-2}}{r^p} - p(2n+1)\frac{x^m y^n}{r^{p+2}} + p(p+2)\frac{x^m y^{n+2}}{r^{p+4}}$$

$$-\ p\frac{x^m y^n}{r^{p+2}} + p(p+2)\frac{x^m y^n z^2}{r^{p+4}}$$

$$\Delta^2\left(\frac{x^m y^n}{r^p}\right) = p\left[(p+2)\cancel{\frac{x^m y^n}{r^{p+2}}} - (2m+1) - (2n+1) - 1\right]\frac{x^m y^n}{r^{p+2}} + m(m-1)\frac{x^{m-2}y^n}{r^p} + n(n-1)\frac{x^m y^{n-2}}{r^p}$$

$$p[p - 2m - 2n - 1]$$

$$l = \frac{m_2x_2 - m_1x_1}{m_1+m_2}$$



$$T = 2\pi\sqrt{\frac{K}{Mgl}} = 2\pi\sqrt{\frac{m_1x_1^2+m_2x_2^2}{(m_1+m_2)g\,\frac{m_2x_2-m_1x_1}{(m_1+m_2)}}}$$

$$= \frac{2\pi}{\sqrt{g}}\sqrt{\frac{m_1x_1^2+m_2x_2^2}{m_2x_2-m_1x_1}} = 2\pi\sqrt{\frac{\lambda_1}{g}}$$

$$\lambda_1 = \frac{m_1x_1^2+m_2x_2^2}{m_2x_2-m_1x_1}$$

$$\lambda_2 = \frac{m_1(\lambda+x_1)^2+m_2(\lambda-x_2)^2}{m_1(\lambda+x_1)+m_2(\lambda-x_2)}$$

Setzt $\lambda_2 = \lambda$

$$\cancel{m_1\lambda^2} + m_1\lambda_2x_1 + \cancel{m_2\lambda^2} - m_2\lambda x_2 = \cancel{m_1\lambda^2} + 2m_1\lambda x_1 + m_1x_1^2 + \cancel{m_2\lambda^2} - 2m_2\lambda x_2 + m_2x_2^2$$

$$m_1x_1^2 + m_1\lambda x_1 + m_2x_2^2 - m_2\lambda x_2 = 0$$

$$\lambda = \frac{m_2x_2^2+m_1x_1^2}{m_2x_2-m_1x_1} = \lambda_1$$

$$\cancel{\lambda_1 - \lambda_2 = \frac{[m_1x_1^2+m_2x_2^2]}{m_2x_2-m_1x_1} - \frac{m_1(\lambda+x_1)^2+m_2(\lambda-x_2)^2}{m_1(\lambda+x_1)+m_2(\lambda-x_2)}}$$

$$\cancel{= m_1^2x_1^2\lambda + m_1^2x_1^3 + m_1m_2x_2^2\lambda + \dots - m_1m_2x_1^2x_2 + m_1m_2\lambda x_2^2 + m_1m_2x_1x_2 + m_2^2\lambda x_2^2 - m_2x_2^3 - m_1m_2x_2(\lambda+x_1)^2 - m_2^2x_2(\lambda-x_2)^2 + m_1^2x_1(\lambda+x_1)^2 - m_1m_2x_1(\lambda-x_2)}$$

$$\lambda - l = \frac{(m_2x_2^2+m_1x_1^2)(m_1+m_2) - (m_2x_2 - m_1x_1)^2}{(m_2x_2-m_1x_1)(m_1+m_2)}$$

$$= m_1m_2x_2^2 + \cancel{m_1^2x_1^2} + \cancel{m_2^2x_2^2} + m_1m_2x_1^2 - \cancel{m_2^2x_2^2} - \cancel{m_1^2x_1^2} + 2m_1m_2x_1x_2$$

$$= m_1m_2(x_1+x_2)^2$$

~~Jüdl~~ $\cancel{\lambda_2 < \lambda}$

$$\cancel{\frac{m_2 x_2^2 + m_1 x_1^2}{m_1 x_1 - m_2 x_2} - \lambda_1 + \lambda < \ell \qquad \lambda < \lambda_1 \qquad m_1 + m_2}$$

$$\lambda - \ell = \frac{m_1 m_2 (x_1+x_2)^2}{(m_2 x_2 - m_1 x_1)(m_1+m_2)}$$

$$\frac{\lambda-\ell}{\ell} = \frac{m_1 m_2 (x_1+x_2)^2}{(m_1+m_2)^2}$$

Jüdl

$$\underline{\underline{\lambda_2 = \lambda(1+\delta)}}$$

$$\frac{m_1(\lambda+x_1)^2 + m_2(\lambda-x_2)^2}{m_1(\lambda+x_1) + m_2(\lambda-x_2)} = \lambda(1+\delta)$$

$$m_1(\cancel{\lambda^2} + 2\lambda x_1 + x_1^2) + m_2(\cancel{\lambda^2} - 2\lambda x_2 + x_2^2) = \cancel{m_1\lambda^2} + \cancel{m_1 \lambda x_1} + \cancel{m_2\lambda^2} - \cancel{m_2 \lambda x_2}$$
$$+ \delta\left[m_1(\lambda+x_1) + m_2(\lambda-x_2)\right]\lambda$$

$$-\lambda + \underbrace{\frac{m_2 x_2^2 + m_1 x_1^2}{m_2 x_2 - m_1 x_1}}_{\lambda_1} = \frac{\delta\left[m_1(\lambda+x_1) + m_2(\lambda-x_2)\right]\lambda}{m_2 x_2 - m_1 x_1}$$

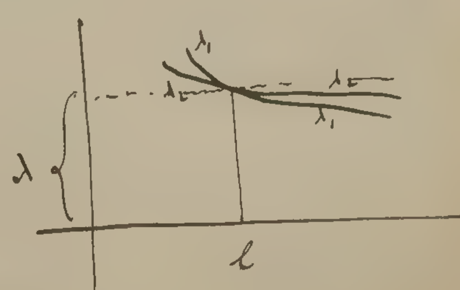
$$\lambda_1 - \lambda = \frac{\lambda\delta\left[m_1(\lambda+x_1) + m_2(\lambda-x_2)\right]}{m_2 x_2 - m_1 x_1} = \lambda\delta\left\{\lambda\frac{(m_1+m_2)}{m_2 x_2 - m_1 x_1} - 1\right\}$$

$$= \lambda\delta\left\{\frac{\lambda}{\ell} - 1\right\}$$

$$\lambda_1 - \lambda = \frac{\lambda_2 - \lambda}{}\left\{\frac{\lambda}{\ell} - 1\right\}$$

$$= (\lambda_2 - \lambda)\left\{\frac{\lambda - \ell}{\ell}\right\}$$

$$= \left[\lambda_2 - \lambda\right]\frac{m_1 m_2 (x_1+x_2)^2}{\ell\,(m_1+m_2)^2}$$



$$\delta = \frac{\lambda_1 - \lambda}{\lambda - \ell}\cdot\frac{\ell}{\lambda} = \frac{\lambda_2 - \lambda}{\lambda}$$

$$\xi_1 = x_1(1+\delta)$$

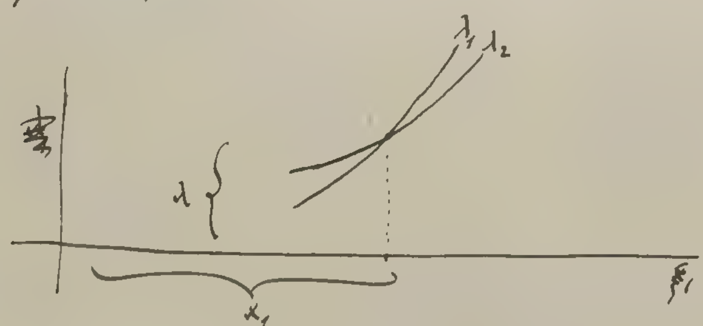
$$\lambda_1 = \lambda + \frac{2m_1x_1^2\delta}{m_2x_2 - m_1x_1} + \frac{m_1x_1\delta}{m_2x_2 - m_1x_1}\lambda$$

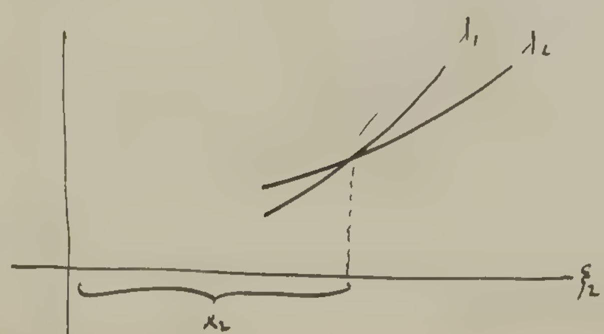
$$= \lambda + \delta\,\frac{m_1x_1}{m_2x_2 - m_1x_1}\left[2x_1 + \lambda\right]$$

$$\lambda_2 = \frac{m_1(\lambda + x_1 + x_1\delta)^2 + m_2(\lambda - x_2)^2}{m_1(\lambda + x_1 + x_1\delta) + m_2(\lambda - x_2)} = \lambda + \frac{2m_1x_1(\lambda + x_1)\delta}{m_1(\lambda + x_1) + m_2(\lambda - x_2)} - \frac{m_1x_1\delta}{m_1(\lambda + x_1) + m_2(\lambda - x_2)}\lambda$$

$$= \lambda + \delta\,\frac{m_1x_1}{m_1(\lambda + x_1) + m_2(\lambda - x_2)}\left[\underbrace{2(\lambda + x_1) - \lambda}_{(\lambda + 2x_1)}\right]$$

ξ_1 zunehmen, x_2 [illegible]





~~[illegible]~~, tues 3 pm

x_1 dane

x_2 zmienne $= \xi_2$

~~$x_2 =$~~ $\xi_2 = x_2(1+\delta)$

$x_1 + l = .$

$$\cancel{\lambda_1 = \lambda(1+\varepsilon)}$$

$$\cancel{\frac{m_2 x_2^2 + m_1 x_1^2}{m_2 x_2 - m_1 x_1} = \lambda(1+\varepsilon)}$$

$$\lambda_1 = \frac{m_1 x_1^2 + m_2 x_2^2 (1+\delta)^2}{m_2 x_2 (1+\delta) - m_1 x_1} = \lambda + \delta \underbrace{\left\{ \frac{2 m_2 x_2^2}{m_2 x_2 - m_1 x_1} - \frac{m_2 x_2}{m_2 x_2 - m_1 x_1} \lambda \right\}}_{\frac{m_2 x_2}{m_2 x_2 - m_1 x_1} \underbrace{\left\{ 2 x_2 - \lambda \right\}}_{\cancel{\frac{m_2 x_2^2 \ldots m_1 x_1 \lambda - m_1 x_1}{m_2 x_2 - m_1 x_1}}}}$$

$\underline{\underline{\xi_2 = x_2(1+\delta)}}$

$$\lambda_1 = \lambda + \delta \cdot \frac{m_2 x_2}{m_2 x_2 - m_1 x_1} \left[2 x_2 - \lambda \right]$$

$$\lambda_2 = \frac{m_1(\lambda + x_1)^2 + m_2(\lambda - x_2 - x_2\delta)^2}{m_1(\lambda + x_1) + m_2(\lambda - x_2 - x_2\delta)} = \lambda - \frac{2 m_2(\lambda - x_2) x_2 \delta}{m_1(\lambda + x_1) + m_2(\lambda - x_2)} + \frac{m_2 x_2 \delta}{m_1(\lambda + x_1) + m_2(\lambda - x_2)} \lambda$$

$$= \lambda - \frac{m_2 x_2 \delta \left[\lambda - 2 x_2 \right]}{m_1(\lambda + x_1) + m_2(\lambda - x_2)}$$

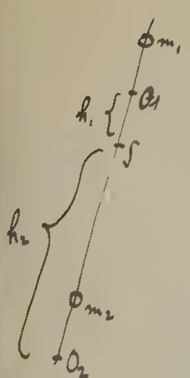
$$\lambda_2 = \lambda + \underline{\delta} \frac{m_2 x_2 \left[2 x_2 - \lambda \right]}{m_1(\lambda + x_1) + m_2(\lambda - x_2)}$$

$$\frac{1}{m_2 x_2 - m_1 x_1} \pm \frac{1}{m_1(\lambda + x_1) + m_2(\lambda - x_2)} = \frac{m_1 \lambda + m_1 x_1 + m_2 \lambda - m_2 x_2 \pm \left[m_2 x_2 - m_1 x_1 \right]}{(\qquad)(\qquad)}$$

$$\lambda_1 + \lambda_2 = 2\lambda + \delta\, m_2 x_2 (2 x_2 - \lambda) \frac{(m_1 + m_2) \lambda}{(m_2 x_2 - m_1 x_1) \left[m_1(\lambda + x_1) + m_2(\lambda - x_2) \right]}$$

$$\lambda_1 - \lambda_2 = \delta\, m_2 x_2 (2 x_2 - \lambda) \frac{m_1 \lambda + 2 m_1 x_1 + m_2 \lambda - 2 m_2 x_2}{(\qquad)\left[\qquad\right]}$$

$$2\lambda = (\lambda_1 + \lambda_2) - (\lambda_1 - \lambda_2) \frac{(m_1 + m_2) \lambda}{(m_1 + m_2) \lambda + 2(m_1 x_1 - m_2 x_2)} = (\lambda_1 + \lambda_2) - (\lambda_1 - \lambda_2) \frac{\lambda}{\lambda - 2l}$$



$$T_1 = 2\pi\sqrt{\frac{K}{Mg\,h_1}} = 2\pi\sqrt{\frac{K_0 + h_1^2 M}{Mg\,h_1}} = \frac{2\pi}{\sqrt{g}}$$

$$\tau = 2\pi\sqrt{\frac{l}{g}}$$

$$g\frac{T_1^2}{4\pi^2} = \frac{K_0}{M}\frac{1}{h_1} + h_1$$

$$g\frac{T_2^2}{4\pi^2} = \frac{K_0}{M}\frac{1}{h_2} + h_2$$

$$\frac{T_1^2}{\tau^2}\,l = \frac{k^2}{h_1} + h_1$$

$$\frac{T_2^2}{\tau^2}\,l = \frac{k^2}{h_2} + h_2 = \frac{k^2}{l-h_1} + l - h_1$$

$$\frac{g}{4\pi^2}\left[h_1T_1^2 - h_2T_2^2\right] = \left[h_1^2 - h_2^2\right] = \frac{[h_1+h_2]}{\tau^2}\left[h_1T_1^2 - h_2T_2^2\right]$$

$$\frac{g}{4\pi^2}\tau^2 = h_1 + h_2 = l \qquad h_1 + h_2 = l \qquad g\frac{\tau^2}{4\pi^2} = h_1 + h_2$$

$$h_2 = l - h_1$$

$$x_1)^2$$

$$\cancel{l(2h_1 - l) = \frac{g}{4\pi^2}\left[h_1T_1^2 - (l-h_1)T_2^2\right]}$$

$$\cancel{h_1\left\{2l + \frac{g}{4\pi^2}(T_1^2 + T_2^2)\right\} = l^2 - g\frac{l}{4\pi^2}T_2^2}$$

$$h_1 - h_2 = \frac{1}{\tau^2}\left[h_1T_1^2 - h_2T_2^2\right]$$

$$\cancel{2h_1 - l = \frac{1}{\tau^2}\left[h_1T_1^2 - (l-h_1)T_2^2\right]}$$

$$\cancel{h_1\left\{2l - \frac{l}{\tau^2}(T_1^2 + T_2^2)\right\} = l^2\left\{1 - \frac{T_2^2}{\tau^2}\right\}}$$

$$\cancel{h_1\left\{2 - \frac{1}{\tau^2}\left[T_1^2 + T_2^2\right]\right\} = l\left\{1 - \frac{T_2^2}{\tau^2}\right\}}$$

$$\tau^2 = \frac{h_1T_1^2 - h_2T_2^2}{h_1 - h_2}$$

$$h_1\left[1 - \frac{T_1^2}{\tau^2}\right] = \left[l - h_1\right]\left[1 - \frac{T_2^2}{\tau^2}\right]$$

$$\frac{T_1^2 + T_2^2}{2} + \frac{h_1T_1^2 - h_2T_2^2}{h_1 - h_2} - \frac{T_1^2h_1}{2} + h_2\frac{T_1^2}{2} - \frac{T_2^2h_1}{2} + \frac{h_2T_2^2}{2}$$

$$\cancel{\left(\frac{T_2}{T_1}\right)^2 = 1 - \frac{h_1}{l-h_1}\left[1 - \left(\frac{T_1}{\tau}\right)^2\right] = \frac{l + h_1\left(\frac{T_1}{\tau}\right)^2 - 2h_1}{l-h_1}} = \frac{T_1^2}{2}(h_2+h_1) + \frac{T_2^2}{2}(h_1+h_2)$$

$$\cancel{\tau^2 = T_1^2 + \frac{h_2}{h_1-h_2}(T_2^2 + T_1^2) = T_1^2 + \frac{h_2}{h_1-h_2}(T_1^2 - T_2^2)}$$

$$h_1T_1^2 + (h_2 - h_1)T_1^2$$

$$\tau^2 = \frac{T_1^2 + T_2^2}{2} + \frac{1}{2}\,\frac{h_1 + h_2}{h_1 - h_2}\left[T_1^2 - T_2^2\right]$$

$$\left[1 - \left(\frac{T_1}{\tau}\right)^2\right]\cancel{\left[1 - \frac{h_2}{h_1}\right] = \frac{T_1^2 + T_2^2}{2} + \frac{1}{2}\frac{h_1+h_2}{h_1 - h_2}\left[1 - \left(\frac{T_2}{\tau}\right)^2\right]}$$

$T_1 = \tau(1+\delta)$

$$T_2^2 = \frac{-\tau^2(h_1 - h_2) + h_1 T_1^2}{h_2}$$

$$= \frac{\cancel{-\tau^2 h_1} + \tau^2 h_2 + \cancel{h_1\tau^2} + 2h_1\tau^2\delta}{h_2}$$

$$= \tau^2\left(1 + \frac{2h_1}{h_2}\delta\right)$$

$$\frac{m_1 x_1^2 + m_2 x_2^2}{m_2 x_2 - m_1 x_1} = \lambda$$

$$m_2 x_2^2 - m_2 x_2 \lambda = -m_1 x_1 \lambda - m_1 x_1^2$$

$$x_2^2 - x_2\lambda = -\frac{m_1}{m_2}(x_1\lambda + x_1^2)$$

$$x_2 = \frac{\lambda}{2} \pm \sqrt{\frac{\lambda^2}{4} - \frac{m_1}{m_2}(x_1\lambda_1 + x_1^2)}$$

motive o ile

$$x_2 = \frac{\lambda}{2} - \sqrt{\frac{\lambda^2}{4} - \frac{m_1}{m_2}(x_1\lambda_1 + x_1^2)}$$

$$\frac{\lambda^2}{4} > \frac{m_1}{m_2}(x_1\lambda_1 + x_1^2)$$

$$\left(\frac{\lambda}{2} - \frac{m_1}{m_2}x_1\right)^2 > x_1^2\left(\frac{m_1}{m_2} + \frac{m_1^2}{m_2^2}\right)$$

$$\frac{\lambda}{2} - \frac{m_1}{m_2}x_1 > x_1\sqrt{\frac{m_1}{m_2} + \left(\frac{m_1}{m_2}\right)^2}$$

$$\frac{\lambda}{2} > x_1\left\{\frac{m_1}{m_2} + \sqrt{\frac{m_1}{m_2} + \left(\frac{m_1}{m_2}\right)^2}\right\}$$

$$> x_1\frac{m_1}{m_2}\left[1 + \sqrt{1 + \frac{m_2}{m_1}}\right]$$

$$m_2(\lambda - x_2)^2 - m_2\lambda(\lambda - x_2) = m_1\lambda(\lambda + x_1) - (\lambda + x_1)^2$$

$$\lambda - x_2 = \frac{\lambda}{2} \pm \sqrt{\frac{\lambda^2}{4} + \frac{m_1}{m_2}\Big[\underbrace{\lambda^2 + \lambda x_1 - (\lambda + x_1)^2}_{-\lambda x_1 - x_1^2}\Big]}$$

$$\left[\frac{\lambda}{2} \pm \sqrt{\frac{\lambda^2}{4} - \frac{m_1}{m_2}(x_1\lambda_1 + x_1^2)}\right] > \frac{m_1 x_1}{m_2}$$

$$(+)\ \sqrt{} > \frac{m_1}{m_2}x_1 - \frac{\lambda}{2}$$

$$\cancel{\frac{\lambda^2}{4}} - \frac{m_1}{m_2}(x_1\lambda + x_1^2) > \frac{\lambda^2}{4} - \lambda\frac{m_1}{m_2}x_1 + \left(\frac{m_1}{m_2}\right.$$

$$(-)\ \frac{\lambda}{2} - \frac{m_1}{m_2}x_1 > \sqrt{\frac{\lambda^2}{4} - \frac{m_1}{m_2}(x_1\lambda + x_1^2)}$$

$$\cancel{\frac{\lambda^2}{4}} - \cancel{\lambda\frac{m_1}{m_2}x_1} + \left(\frac{m_1 x_1}{m_2}\right)^2 > \cancel{\frac{\lambda^2}{4}} - \cancel{\lambda\frac{m_1}{m_2}x_1} - \frac{m_1}{m_2}x_1^2$$

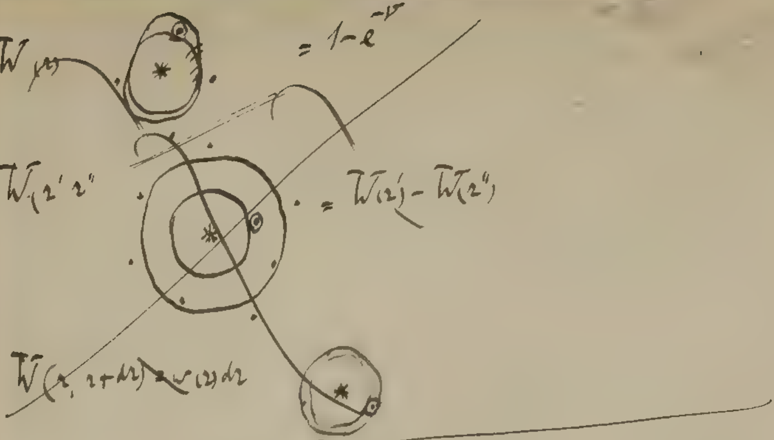
wenn man wählt $h_2 = 2h_1$, so ist

$$\tau^2 = \frac{h_1 T_1^2}{h_1} = T_1^2$$

$W_{(r)}$ $= 1-e^{-\nu}$

$W_{(r'\,r'')}$ $= W(r') - W(r'')$

$W_{(r,\,r+dr)} = w(r)\,dr$



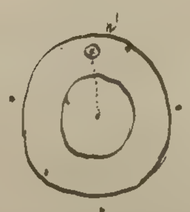
$W_{(r)}$

$= 1-e^{-\nu}$ = Chance 1, 2, 3 Nachbarn

$= \sum \nu e^{-\nu} + \nu^2 \frac{e^{-\nu}}{2} + \dots$

= Chance wenigstens 1 Nachbar

$W_{(r'\,r'')}$



$= W(r') - W(r'')$ ~~ergo nicht [illegible]~~ $\cancel{= W(r') (1 - W(r''))}$

$W_{(r,\,r+dr)} = w(r)\,dr = \frac{\partial W}{\partial r}\,dr = e^{-\nu}\frac{d\nu}{dr}\,dr = 4\pi r^2 \cdot e^{-\nu}\,dr\,\underline{n}$

$W_{(r+dr)}[1 - W(r)] =$

Wahrsch. dass nur einer innerhalb r_1 aber gar keiner innerhalb r_2:

$\cancel{\frac{4\pi}{3} n r^3 e^{-\frac{4\pi}{3}[(r+dr)^3 - r^3] n}}$

$= \frac{4\pi}{3} n [(r+dr)^3 - r^3]\, e^{-\frac{4\pi n}{3} r^3} = 4\pi r^2 n\,dr\,e^{-\frac{4\pi n r^3}{3}}$

Für Wahrsch. dass nur einer zwischen $r_1 r_2$ vorkommt, kann man nicht sagen: $W r_1 = W(r_1, r_2) + W(r_2)$

sondern $W(r_1) = \dots\, e^{-\frac{4\pi}{3}(r_1^3 - r_2^3) n} + \frac{4\pi}{3} n (r_1^3 - r_2^3)\, e^{-\frac{4\pi}{3} n (r_1^3 - r_2^3)} \cdot e^{-\frac{4\pi}{3} n r_2^3}$

$\cancel{\frac{4\pi}{3} n r_2^3 e^{-\frac{4\pi}{3} n r_2^3}}$

$= \frac{4\pi}{3} n r_1^3\, e^{-\frac{4\pi}{3} n r_1^3}$ stimmt

POLSKIE
TOWARZYSTWO PRZYRODNIKÓW
IM. KOPERNIKA.

Lwów dnia . . . .

$$e^{-nk_1} - e^{-nk_2} \geq n(k_2-k_1)\, e^{-nk_2}$$

$$e^{-n(k_1-k_2)} \geq n(k_2-k_1) + 1$$

$$e^{n(k_2-k_1)} > 1 + n(k_2-k_1)$$

$$e^{-nk_1}$$

$$\frac{d(e^{-nk_1}}{dn}\, dn$$

$$e^{-nk}$$

$$\lim_{\sigma=0} \psi = -\frac{a^2\alpha \sin^2\theta}{2}\Bigg[\left\{\cos(\sigma t + \frac{3}{2\beta a}(\cos\sigma t + \sin\sigma t) + \frac{3}{2\beta^2 a^2}\sin\sigma t\right\}\frac{a}{r}$$

$$-\left\{\frac{3}{2\beta a}\left[\cos(\sigma t - \beta(r-a)) + \sin(\sigma t - \beta(r-a))\right] - \frac{3}{2\beta^2 a^2}\sin(\sigma t - \beta(r-a))\right\}\left\{1 - \beta(r-a) + \frac{\beta^2(r-a)^2}{2}\right\}\Bigg]$$

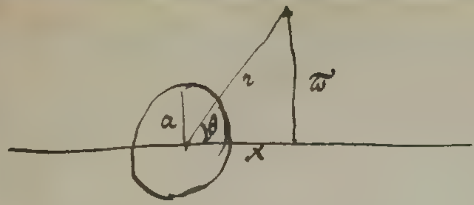
$$\beta = \sqrt{\frac{\sigma\rho}{2\mu}}$$

$$= -\frac{3\alpha\sin^2\theta}{4\beta^2}\Bigg[\left\{\sin\sigma t + \beta a(\cos\sigma t + \sin\sigma t) + \frac{2\beta^2 a^2}{3}\cos\sigma t\right\}\frac{a}{r}$$

$$-\underbrace{\left\{\sin(\sigma t - \beta(r-a)) + \beta a\cos(\sigma t - \beta(r-a))\right\}}\left\{1 - \beta(r-a) + \frac{\beta^2(r-a)^2}{2}\right\}\Bigg]$$

$$\sin\sigma t\cos\beta(r-a) - \cos\sigma t\sin\beta(r-a)$$

$$+\cos\sigma t\cos\beta(r-a) + \sin\sigma t\sin\beta(r-a)$$



$$u = -\frac{1}{\varpi}\frac{\partial\psi}{\partial\varpi} \qquad V = \frac{1}{\varpi}\frac{\partial\psi}{\partial x}$$

$$\psi = -\frac{1}{2}\alpha a^2 \sin^2\theta\left[\left\{\left(1+\frac{3}{2\beta a}\right)\cos(\sigma t+\varepsilon) + \frac{3}{2\beta a}\left(1+\frac{1}{\beta a}\right)\sin(\sigma t+\varepsilon)\right\}\frac{a}{r}\right.$$

$$\left. - \frac{3}{2\beta a}\left\{\cos[\sigma t-\beta(r-a)+\varepsilon] + \left(1+\frac{1}{\beta r}\right)\sin[\sigma t-\beta(r-a)+\varepsilon]\right\}e^{-\beta(r-a)}\right]$$

$$U = \alpha\cos(\sigma t+\varepsilon)$$

$$\beta = \sqrt{\frac{\sigma\rho}{2\mu}}$$

$$= \psi_1 + \psi_2$$

$$\Phi_1 = -4\mu\left[\frac{\partial v}{\partial y}\frac{\partial w}{\partial z} - \frac{\partial v}{\partial z}\frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}\frac{\partial u}{\partial x} - \frac{\partial w}{\partial x}\frac{\partial u}{\partial z} + \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\right] = \quad \overset{1=}{(\text{irrotational})}$$

$$= -4\mu\left[\frac{\partial^2\varphi}{\partial y^2}\frac{\partial^2\varphi}{\partial z^2} - \left(\frac{\partial^2\varphi}{\partial y\partial z}\right)^2 + \frac{\partial^2\varphi}{\partial z^2}\frac{\partial^2\varphi}{\partial x^2} - \left(\frac{\partial^2\varphi}{\partial x\partial z}\right)^2 + \frac{\partial^2\varphi}{\partial x^2}\frac{\partial^2\varphi}{\partial y^2} - \left(\frac{\partial^2\varphi}{\partial x\partial y}\right)^2\right] =$$

$$= 4\mu\left[\left(\frac{\partial^2\varphi}{\partial x^2}\right)^2 + \left(\frac{\partial^2\varphi}{\partial y^2}\right)^2 + \left(\frac{\partial^2\varphi}{\partial z^2}\right)^2 + 2\left(\frac{\partial^2\varphi}{\partial y\partial z}\right)^2 + 2\left(\frac{\partial^2\varphi}{\partial x\partial z}\right)^2 + 2\left(\frac{\partial^2\varphi}{\partial x\partial y}\right)^2\right]$$

$$v = V\frac{y}{\varpi} = V\frac{y}{\sqrt{y^2+z^2}} = \cancel{}\ \frac{y}{y^2+z^2}\frac{\partial\psi}{\partial x}$$

$$w = V\frac{z}{\sqrt{y^2+z^2}} = \frac{z}{y^2+z^2}\frac{\partial\psi}{\partial x}$$

hence $\varphi = \boxed{A}\frac{x}{r^3}$

$$\frac{\partial\varphi}{\partial x} = \frac{1}{r^3} - \frac{3x^2}{r^5} \qquad \frac{\partial^2\varphi}{\partial x^2}$$

$$\frac{\partial\varphi}{\partial y} = \quad - \frac{3xy}{r^5}$$

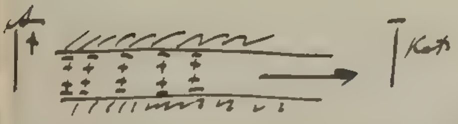
$$\frac{\partial\varphi}{\partial z} = \quad - \frac{3xz}{r^5}$$

$$\Phi_1 = 4\mu\left[\left[-\frac{9x}{r^5} + \frac{15x^3}{r^7}\right]^2 + \left[-\frac{3x}{r^5} + \frac{15xy^2}{r^7}\right]^2 + \left[-\frac{3x}{r^5} + \frac{15xz^2}{r^7}\right]^2 + 2\left(\frac{15xyz}{r^7}\right)^2 + 2\left(-\frac{3z}{r^5} + \frac{15x^2z}{r^7}\right)^2\right.$$

$$Vol = \frac{K(\varphi_1 - \varphi_0)}{4\pi} \, \frac{J\sigma}{\mu}$$

	J
$-SiO_2 + H_2O$ (Sol)	$> 4.7 \cdot 10^{-7}$ Am.
Sb_2O_5	> 6.4 "
$WO_3 \cdot H_2O$	8.0 "
$SnO_2 + H_2O$	$11. \ 10^{5}$
SnO_2	$8.$ "
$TiO_2 + H_2O$	1 "
Fe_2O_3	$7.3 \ 10^{-11}$ NaOH
$Fe(OH)_3$	5.6
CuO	4.6
$Cu(OH)_2$	< 2.9
ZnO	2.8
$Zn(OH)_2$	1.3
$Ni(OH)_2$	$1.5 \cdot 10^{-13}$
$Mg(OH)_2$	< 8.6 "
MgO	$1.5 \cdot 10^{-14}$

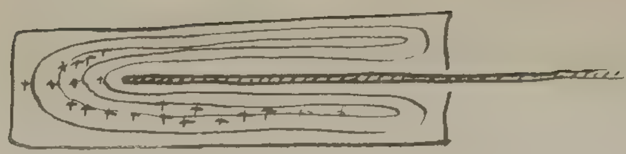
$$v = A - K \log p$$

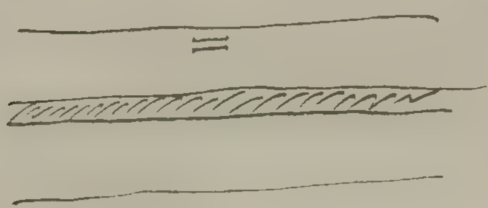


H_2WO_4		$v = +2.66$
$+HCl$	0.0062	$+2.41$
	0.045	$+1.12$
	0.207	$+0.28$
	0.8	0.0
	(Mol)	

ZnO	(Mol)	$v = -0.31$
NaOH	0.0002	-0.22
	0.0003	$+0.16$
	0.0004	$+0.24$

$Mg(OH)_2$				
	0.0006	$v = -0.64$	0.015	$+0.65$
	0.005	-0.23	0.012	$+0.99$
	0.01	-0.05	$+0.009$	$+0.32$
	0.011	-0.10		
	0.02	$+0.90$		





$$-\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{rq}{\omega\varepsilon} = 0$$

$$d\left(r\frac{dV}{dr}\right) = r\frac{q}{\omega\varepsilon}$$

$$r\frac{dV}{dr} = \frac{r^2}{2}\frac{q}{\omega\varepsilon} + \alpha$$

$$V = \frac{r^2}{4}\frac{q}{\omega\varepsilon} + \alpha \lg r + \beta$$

$$V_1 = \frac{r_1^2}{4}\gamma + \alpha \lg r_1 + \beta$$

$$V_2 = \frac{r_2^2}{4}\gamma + \alpha \lg r_2 + \beta$$

$$V_1 - V_2 = \frac{r_1^2 - r_2^2}{4}\gamma + \alpha \lg \frac{r_1}{r_2}$$

q = liczba jonów $_+$ pro sek. wytworzonych

$\varepsilon\omega$ = prędkość jonu w polu $\frac{1\,\text{Volt}}{1\,\text{cm}}$

$$V - V_2 = \frac{(r^2 - r_2^2)}{4}\gamma + \alpha \lg \frac{r}{r_2}$$

$$= \frac{r^2 - r_2^2}{4}\gamma + \left\{(V_1 - V_2) - \frac{r_1^2 - r_2^2}{4}\gamma\right\} \frac{\lg r - \lg r_2}{\lg r_1 - \lg r_2}$$

$$= (V_1 - V_2)\frac{\lg r - \lg r_2}{\lg r_1 - \lg r_2} +$$

$$\omega\varepsilon\left(r\frac{\partial V}{\partial r}\right)_1 = \omega\varepsilon\left[\frac{r_1^2}{2}\frac{q}{\omega\varepsilon} + \alpha\right] = \frac{r_1^2}{2}q + \alpha\,\omega\varepsilon$$

$$= \frac{r_1^2 q}{2} + \frac{V_1 - V_2 - \frac{r_1^2 - r_2^2}{4}\gamma}{(\lg r_1 - \lg r_2)}\,\omega\varepsilon$$

$$e^{-\frac{v}{2}\delta^2 \cdot \frac{v_0^2}{R\theta_0}\left(\frac{\partial p}{\partial v}\right)_0}$$

$$e^{-\frac{v\delta^2}{2} - \frac{\Delta U}{\frac{R\theta}{2}}}$$

$$pv = R\theta - \frac{1}{2}\Sigma r f(r)$$

$$\frac{mu^2}{2} = \frac{m}{2}c^2 = \frac{R\theta}{2}$$

$$v\frac{\partial p}{\partial v} + p = -\frac{1}{2}\frac{\partial}{\partial v}\left[\Sigma r f(r)\right]_{n=\mathrm{const}}$$

$$v^2\frac{\partial p}{\partial v} + pv = -\frac{1}{2}v\frac{\partial}{\partial v}\left[\Sigma r f(r)\right]$$

$$\Sigma r f(r) = \Sigma \iint r f(r)\, e^{-hU(x,y,\ldots)} dx \ldots$$

$$\Sigma(xX + yY + zZ)\, e^{-hU(x,y,z)}\, dx\, dy\, dz$$

$$\frac{v^2\frac{\partial p}{\partial v}}{R\theta} + \frac{pv}{R\theta} = = \frac{\frac{1}{2}v\frac{\partial}{\partial v}(\Sigma r f)}{R\theta}$$

$$\frac{v^2\left(\frac{\partial p}{\partial v}\right)}{R\theta} = \frac{1}{2}\,\frac{\overbrace{\Sigma\Sigma r f(r)}^{V} - v\frac{\partial}{\partial v}\Sigma r f(r)_{n=\mathrm{const}}}{R\theta} - 1$$

$$V - \frac{v}{2}\frac{\partial}{\partial v}V$$

$$\cancel{= \frac{1}{v}\frac{\partial}{\partial v}(vV)}$$

$$p\,\mathrm{vol} = R\theta - (\overbrace{\Sigma\Sigma r f(r)}^{V})$$

$$p_0\,\mathrm{vol} = R\theta = V_0$$

$$\frac{\partial p}{\partial v}\cdot \mathrm{vol} = -\frac{\partial V_{n,v}}{\partial v}\Big|_{\mathrm{vol}=\mathrm{const}}$$

$$v^2\frac{V\,dv - v\,dV}{v^2\,dv}$$

$$= -v^2\frac{d}{dv}\left(\frac{V}{v}\right)$$

$$v\left(\frac{V}{v} - \frac{1}{v}\frac{\partial V}{\partial v}\right) = v\frac{\partial}{\partial v}\left(\frac{V}{v}\right)$$

$$\frac{\Delta V_{n=\mathrm{const}}}{} = \frac{\Delta V_{\mathrm{vol\,const}}}{\partial v} + V\frac{\partial}{\partial n}\Delta n$$

$$= -\frac{1}{\rho^2}\,\frac{d(V\rho)}{d\left(\frac{1}{\rho}\right)} = = -\frac{d(V\rho)}{\rho}$$

$$\int (p - p_0)\,dv = R\theta\left[\log\frac{v}{v_0} - \frac{v - v_0}{v_0}\right] -$$

$$\Sigma r\,(\ldots) - \Sigma r_0\ldots$$

$$\int_1^2 r\frac{\partial U}{\partial r}\,dr = (Ur)\Big|_1^2 - \int_1^2 U\,dr$$

$$- r_1\left(\frac{\partial U}{\partial r}\right)_1 (r_2 - r_1)$$

$$\Sigma\Sigma\int_1^2 r\frac{\partial U}{\partial r}\,dr - r_1\left(\frac{\partial U}{\partial r}\right)_1 (r_2 - r_1)$$

$$U\Delta r + r\Delta U - \cancel{U\Delta r} - r\underbrace{\frac{\partial U}{\partial r}\Delta r}_{\Delta U} = 0$$

$$\frac{\partial U(r)}{\partial r}\Delta r + \boxed{\frac{\partial^2 U(r)}{\partial r^2}\Delta r^2} - -$$

$$\Delta r^2\cdot\left[2\frac{\partial U}{\partial r} + r\frac{\partial^2 U}{\partial r^2}\right]$$

$t = 10$ sek.

1). $c = 50 \frac{m}{sek.}$ $t = 5$

$\frac{100}{2} 10 = 500$ m.

$p = -\frac{\partial F}{\partial v}$

$p_0 = -\left(\frac{\partial F}{\partial v}\right)_0$

$\int (p - p_0)\, dv = F_{F_0} - \left(\frac{\partial F}{\partial v_0}\right)(v - v_0)$

$U = F - \theta \frac{\partial F}{\partial \theta}$

$\frac{\partial U}{\partial v} = \frac{\partial F}{\partial v} - \theta \frac{\partial^2 F}{\partial \theta \partial v} = p - \theta \frac{\partial p}{\partial \theta}$

$\delta U = \frac{U_{-1} + U_{+1} - 2U_0}{2} = \frac{\partial^2 U}{\partial v^2} \delta v^2$

$\frac{\partial^2 U}{\partial v^2} = \frac{\partial p}{\partial v} - \theta \underbrace{\frac{\partial}{\partial \theta}\left(\frac{\partial p}{\partial v}\right)}_{v\delta^2\ ??}$

$p + \frac{a}{v^2} = \frac{R\theta}{\pi}\left[\frac{1}{v} + \frac{b}{v^2} + \frac{5}{8}\frac{b^2}{v^3}\right]$

$\frac{\partial p}{\partial v} = -\frac{2a}{v^3} - R\theta\left[\frac{1}{v^2} + \frac{2b}{v^3} + \frac{15}{8}\frac{b^2}{v^4}\right]$

$-\frac{v^2}{R\theta}\left(\frac{\partial p}{\partial v}\right) = 1 + \frac{2b}{v} + \frac{15}{8}\frac{b^2}{v^2} + \frac{2a}{v . R\theta}$

$\frac{v\delta^2}{2}\left[\frac{v_0^2}{R\theta_0}\left(\frac{\partial p}{\partial v}\right)_0\right]$

$b = 4\left[\frac{4}{3}\pi\left(\frac{\sigma}{2}\right)^3\right] = \frac{2}{3}\pi\sigma^3 v$

$2b = \alpha v$ $= \frac{\alpha v}{2}$

$(2 . 10^{-5})^3 . 3 . 10^{19}$

$v = 24 . 10^5$

$\sqrt{v} = 5 . 10^2$

$\frac{1}{500}$

$h = \frac{3N}{2L} = \frac{3N}{2N\frac{mc^2}{2}} = \frac{3}{mc^2}$

$\frac{mnc^2}{3} = R\theta$

$\frac{3}{mc^2} = \frac{n}{R\theta} = \frac{n\mu}{H\theta} = \frac{nm}{mR\theta}$

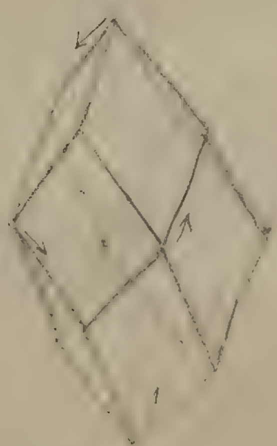
$h = \frac{V}{H\theta}$

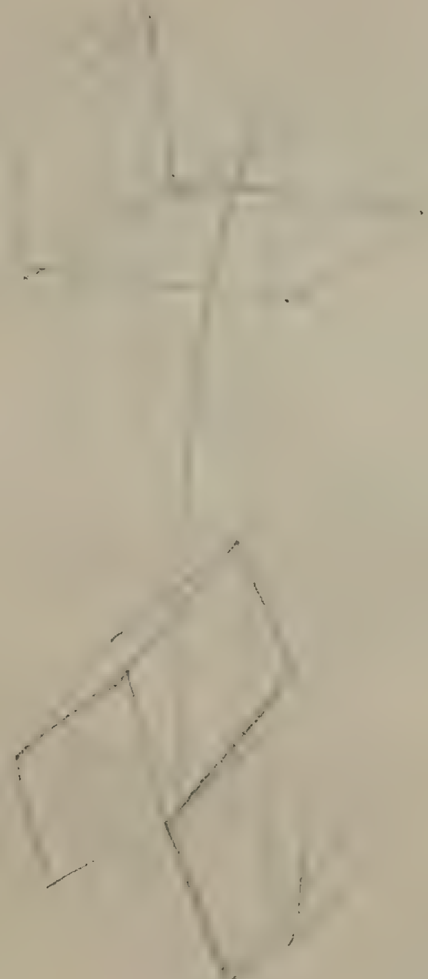
$\frac{1}{\sqrt{3 . 10^{19}}}$

$= \frac{10^{-9}}{\sqrt{30}}$

$= \frac{1}{5}$

$= 2 . 10^{-10}$





Dlatego tylko 100 km anizotropii, bo ciśnienie photonowe tylko w tej

2). Głębokość epicentrum Rudzki Kalabria 1905 7 km
S. Francisco 1906 16–32

Helmert Hayford Ausgleichsfläche, poniżej której już ciężary zgodnie z materiałem nie wpływają
stałe warstwy plastyczne

sprawdzić ciało stałe, bo inaczej nie byłoby poprzecznych, ale
skorupa spoczywa na głębszych warstwach o wyższym stopniu plastyczności

---

Teorya izostazyi równowaga hydrostatyczna gór lżejszych

przykład Rudzkiego: Norwegia 280 km – ląd mocno podniesiony po epoce lodowej

dostateczny pod lód 933 m
7 km

ale nie masa płynna jak woda tylko plastyczna; prawie teorya dobra! Ruch 3–40,000 lat
Kanada, Chicago-Detroit

Wiek ziemi

Kelvin Rudzki tylko 139–486 mil lat
Kew 40–100 dostatecznie wszystko promieniotwórczości

Strutt: He		Holmes U→Pb	
oligocen	8	węgl.	340
eocen	31	sylur	430
wczesny węgl.	150	arch.	1025–1640
arch.	710		

Dotychczas tylko fizyczne teorye z zakresu własności geofizyki
fale wodne Helmholtz

działa { ~~[illegible]~~ Fizyka ziemi
meteorologia

geofizyka : siła przyrody
dążenie do ścisłości sformułowania | niewiadomości ... geologia, studium matematyczne
fizyka
geofizyka stwierdza ...

Słowo o epoce lodowej

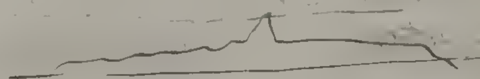
charakterystyczny ... wobec ... , ... tylko w tym co daje się ...

Dalszego trudno ... ...

ograniczę się do ogólnikowego poglądu na ... zagadnienie

1). wewnętrzny kształt kuli ziemskiej

na pozór rzecz prosta. Gemont 1900 m – co to znaczy?



Stokes, obliczenie teoretyczne

Helmert metoda kondensacji, Rudzki inny sposób

związek z ~~...~~ wahadłem ciężkości, z anomaliami

2). Seismologia

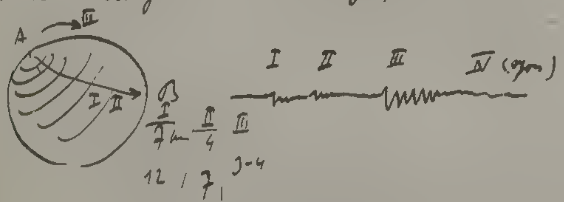
najpoważniejszy materiał do poznania wnętrza ziemi, tak jak prześwietlenie

30 – 40.000 trzęsień rocznie, 100 stacji, ... tylko rejestrowanie!

Fakty znane : góry, ... spręż. objętości, fale objętości, ...

ciało stałe izotropowe, 2 ... 2 fale   oraz fale Rayleigh – Lamb
wzdłuż powierzchni jak fale wodne

analiza sejsmogramów : 3 rodzaje fal



Badania Rudzkiego : 1). skały skorupy anizotropowe (Sneis, Tyrol) ... jednorodnego

3 fale  analogia z podw. załamaniem

Stanowisko to ... przewrót, rozwiązuje, gdyby całe kula ziemska złożona

Tymczasem (tylko 100 – 120 km. , 6270 km. ...
Rudzki sam.)

odwrotnego pierwiastku z gęstości (proporcyonalny do N), podczas gdy szybkość opadania zostaje niezmienioną, a szybkość prądów związanych z różnicami temperatury (musi się zmniejszyć) w stosunku gęstości.

Zauważę też że obserwacye kondensacyi pary na jonach (Wilson, Thomson etc.), i związane z tem zjawiska elektryczne (mogą też ciekawego materyału dostarczyć. a co
umówić się prosz!

---

§15). Znacznie większe trudności przedstawia zastosowanie tych rozumowań do cieczy, ponieważ wogóle mało jeszcze wiemy o mechanizmie ich ruchów cząsteczkowych. Do obliczenia liczby uderzeń n nie możemy użyć owego wzoru z teoryi gazów (16) ważnym jest tylko dla gazu rozrzedzonego. Można w każdym razie stawić

$$n = 4\pi R^2 \nu \qquad (19)$$

a zatem

$$\Delta = \frac{2\sqrt{2}}{3\sqrt{\pi}}\,\frac{c}{R\sqrt{\nu}} \qquad (20)$$

gdzie ν oznacza ilość uderzeń na 1 cm² powierzchni, ale pytanie znów jak oznaczyć ν. Podamy kilka sposobów mniej lub więcej nieścisłych, które jednak co do rzędu tej wielkości ν mogą dać wskazówki.

a). Według Van der Waalsa istnieje w cieczy wewnętrzne ciśnienie $p_i = a\rho^2$ które obliczyć można z (ciepła wewnętrznego) parowania L_i (t.j. z ciepła parowania pozostającego po odciągnięciu pracy zewnętrznej) na podstawie wzoru: $L_i = a\rho$

Owo ciśnienie (wywierane na powierzchnię wewnętrzną) jest równoważone przez uderzenia drobin, z których każde reprezentuje przeciętnie moment ruchu $2mc\int_0^{\frac{\pi}{2}}\sin\theta\cos\theta\, d\theta = mc$; zatem wynika równanie:

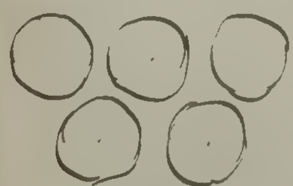
$$\nu = \frac{p_i}{mc} = \frac{L_i\rho}{mc} \qquad (21)$$

~~Podstawiając B w λ otrzymujemy:~~

$$\cancel{\lambda = \frac{2\sqrt{2}}{3\sqrt{\pi}} \frac{c^{3/2}}{R} \sqrt{\frac{m}{\mu}}}$$

b). Spółczynnik lepkości (~~z punktu widzenia teoryi kinetycznej~~) jest to ilością ruchu udzieloną na sekundę płaszczyźnie XZ nieruchomej przez drobiny o nią uderzające, jeżeli spad prędkości $\frac{\partial u}{\partial y}$ (równoległej do X) ~~wynosi~~ równa się jedności.

Otrzymamy ten spółczynnik mnożąc ilość uderzeń przez przeciętną ich ilość ruchu. Gdyby drobiny były punktami, wtedy (te które) ~~[illegible]~~ pod kątem θ ~~[illegible]~~ ~~[illegible]~~ ~~[illegible]~~ uderzają,



przechodziłyby z warstwy odległej o $\lambda \cos\theta$ od płaszczyzny XZ, jeżeli λ jest ich drogą swobodną, i posiadałyby ilość ruchu $\lambda \cos\theta$. Z powodu rozmiarów drobin trzeba jednak dodać do λ ~~[illegible]~~ średnicę 2r dla uderzeń prostych, a pewien ułamek tej wielkości dla skośnych uderzeń. Ponieważ chodzi nam tylko o rząd wielkości, nie wdamy się w dokładne obliczenie tego ułamka, tylko stawiamy $\lambda+2r$ na miejsce λ. Zatem mamy

$$\nu m (\lambda + 2r) \int_0^{\frac{\pi}{2}} \cos\theta \sin\theta \, d\theta = \mu$$

Rozmaite względy wskazują ~~Powinniśmy [illegible] przyjąć~~ że w cieczach przestrzeń wolna dla ruchu jest małym ułamkiem przestrzeni zajętej, więc ~~[illegible]~~ pominiemy λ w porównaniu z 2r i otrzymamy:

$$\nu = \frac{\mu}{r m} \qquad (22)$$

c). ~~W przeciwieństwie [illegible]~~ Wzór dla spółczynnika dyfuzyi $D = \frac{\lambda c}{3}$ (patrz powyżej) pod pewnemi zastrzeżeniami także dla cieczy będzie przybliżenie ważnym. ~~Otrzymamy~~ Spółczynnik dyfuzyi wody przez wodę [illegible]

$$+2\left(-\frac{3y}{r^5}+\frac{15x^2y}{r^7}\right)^2\Big]$$

$$=4\mu\cancel{\left(\frac{3x}{r^5}\right)^2\left(3+5\frac{x^2}{r^2}\right)^2}$$

$$=4\mu\left(\frac{3x}{r^5}\right)^2\left\{\left(3-5\frac{x^2}{r^2}\right)^2+\left(1-5\frac{y^2}{r^2}\right)^2+\left(1-5\frac{z^2}{r^2}\right)^2\right\}+2\cdot\frac{9z^2}{r^{10}}\left(1-\frac{5x^2}{r^2}\right)^2$$

$$\underbrace{\cancel{1-10-2\ldots\cos^2\alpha+25(\cos^4\alpha+\cos^4\beta+\cos^4\gamma)}}_{\left[1-10\frac{y^2+z^2}{r^2}+25\frac{(y^2+z^2)^2}{r^4}\right]}\qquad+2\cdot\frac{9y^2}{r^{10}}\left(1-\frac{5x^2}{r^2}\right)^2\Big]$$

$$=\left(\frac{3x}{r^5}\right)^2\left\{\underbrace{\left[3-5\frac{x^2}{r^2}\right]^2+\left[1-10\left(1-\frac{x^2}{r^2}\right)+25\left(1-\frac{x^2}{r^2}\right)^2\right]}\right\}+18\left(1-\frac{5x^2}{r^2}\right)^2\left(1-\frac{x^2}{r^2}\right)\frac{1}{r^8}$$

$$\cancel{9}-30\frac{x^2}{r^2}+25\frac{x^4}{r^4}+\cancel{1}-\cancel{10}+10\frac{x^2}{r^2}+25-50\frac{x^2}{r^2}+25\frac{x^4}{r^4}\Big]$$

$$1-10\frac{x^2}{r^2}+25\frac{x^4}{r^4}$$

$$-\frac{x^2}{r^2}+10\quad-25$$

$$=\frac{9}{r^8}\left\{\frac{x^2}{r^2}\left[25-70\frac{x^2}{r^2}+50\frac{x^4}{r^4}\right]+2\left[1-11\frac{x^2}{r^2}+35\frac{x^4}{r^4}-25\frac{x^6}{r^6}\right]\right\}$$

$$=\frac{9}{r^8}\left\{2+3\frac{x^2}{r^2}\right\}$$

$$\int\cos^2\vartheta\sin\vartheta\,d\vartheta=-\frac{\cos^3}{3}\Big|_0^\pi=\frac{2}{3}$$

$$\Phi_1=36\frac{\mu}{r^8}\left[2+3\frac{x^2}{r^2}\right]A^2\qquad A=\frac{\alpha a^3}{2}\left[\left(1+\frac{3}{2\beta a}\right)\cos\ldots+\frac{3}{2\beta a}\left(1+\frac{1}{\beta a}\right)\sin\ldots\right]$$

$$\Sigma\Phi_1=\iint_{\theta=0}^{\pi}2\pi r^2\sin\theta\,d\theta\,dr\ \Phi_1=2\pi\cdot36\mu\,A^2\cdot\left(\frac{1}{5a^5}-\frac{1}{5r^5}\right)\left[4+3\cdot\frac{2}{3}\right]=\frac{2.36.6.\pi\mu A^2}{5.a^5}$$

$$T = \frac{\mu}{2}\left[\dot y_1^2 + \dot y_2^2 + \dots \dot y_n^2 + \dot y_{n+1}^2 + \dot y_{n+2}^2\right]$$

$$U = \frac{T_1}{2a}\left[(y_2-y_1)^2+(y_3-y_2)^2+\dots (y_n-y_{n+1})^2+(y_{n+2}-y_{n+1})^2\right] \qquad y_1 = y_{n+2} = 0$$

$$O y_1 + A y_2 + O y_3 = 0$$

$$O y_2 + A y_3 + O y_4 = 0$$

$$\vdots$$

$$\cancel{\mu \ddot y_1 = + \frac{T_1}{a}(y_2 - y_1)}$$

$$\mu \ddot y_2 = -\frac{T_1}{a}\left[(y_2-y_1)-(y_3-y_2)\right]$$

$$\mu \ddot y_2 + \frac{2T_1}{a} y_2 = \frac{T_1}{a}(y_1+y_3)$$

$$y_m = \sum_{s=1}^{n} P_s \sin\frac{(m-1)s\pi}{n+1} \cos(n_s t - \varepsilon_s)$$

$$n_s = 2\sqrt{\frac{T_1}{\mu a}} \sin\frac{s\pi}{2(n+1)} \qquad s = 1 \dots n$$

$s=1$: $\quad y_m = P_1 \sin\frac{m-1}{n+1}\pi \cos\dots$

$y_1 = 0$

$$\left.\begin{aligned} y_2 &= P_1 \sin\frac{\pi}{n+1} \\ y_3 &= P_1 \sin\frac{2\pi}{n+1} \\ y_4 &= P_1 \sin\frac{3\pi}{n+1} \\ &\vdots \end{aligned}\right\} \cos\dots$$

$$y_2 = \sin\frac{2\pi}{n+1}, \quad y_3 = \sin\frac{4\pi}{n+1}, \quad y_4 = \sin\frac{6\pi}{n+1}$$

$$y_2 = \sin\frac{3\pi}{n+1}, \quad y_3 = \sin\frac{6\pi}{n+1}, \quad y_4 = \sin\frac{9\pi}{n+1}$$

$$q_1 = \alpha_{11} y_1 + \alpha_{12} y_2 + \alpha_{13} y_3 + \dots$$

$$q_2 = \alpha_{21} y_1 + \alpha_{22} y_2 + \alpha_{23} y_3 \dots$$

$$q_3 \dots$$

$$y_1 = \beta_{11} q_1 + \beta_{12} q_2 + \dots$$

$$y_2 = \beta_{21} q_1 + \beta_{22} q_2 + \dots$$

$$\vdots$$

$$q_s = \frac{2}{n+1} \sum_{r=1}^{n} y_r \sin\frac{r s \pi}{l}$$

#

$$\begin{vmatrix} c & 1 & 0 & 0 & 0 \\ 1 & c & 1 & 0 & 0 \\ 0 & 1 & c & 1 & 0 & \cdot \\ 0 & 0 & 1 & c & 1 & 0 \end{vmatrix} = 0$$

$$\left\{ \begin{array}{l} \varphi_1 = 0 \\ \varphi_{m+1} = 0 \end{array} \right.$$

$$\varphi_r = B \sin (r-1)\beta \cos(nt - \varepsilon) \qquad \underline{\underline{(m+1)\beta = s\pi}}$$

$$B \sin(r-2)\beta + A \sin(r-1)\beta + B \sin r\beta = 0$$

$$B \underbrace{[\sin(r-2)\beta + \sin r\beta]}_{2 \sin(r-1)\beta \cdot \cos\beta} + A \sin(r-1)\beta = 0$$

$$2 B \cos\beta + A = 0$$

$$\frac{A}{B} = -2 + \frac{\mu a n^2}{T} = -2 \cos\beta$$

$$n = 2 \sin \frac{\beta}{2} \sqrt{\frac{T}{\mu a}}$$

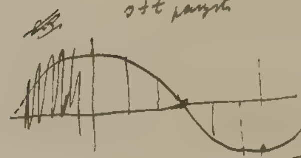
$$y_r = \sum_{s=1}^{m} P_s \sin\frac{(r-1)s\pi}{m+1}\cos(n_s t-\epsilon) \qquad = \sum \alpha q$$

$$n_s = 2\sqrt{\frac{T_1}{ma}}\,\sin\frac{s\pi}{2(m+1)}$$

$$\sum_{s=1}^{m} \sin^2\frac{s\pi x}{l} = \frac{m+1}{2}$$

$$\underbrace{\sum_{s=1}^{m} \sin\frac{s\pi x}{l}\sin\frac{t\pi x}{l}}_{\cos(s+t)\frac{x\pi}{l} - \cos(s-t)\frac{x\pi}{l}} = 0$$

$s+t$ [illegible]



$$q_s = \sum_{r=1}^{m} y_r \sin\frac{(r-1)s\pi}{m+1}$$

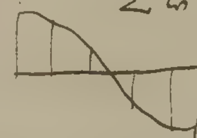
$$\dot q_s = \sum_{r=1}^{m} \dot y_r \sin\frac{(r-1)s\pi}{m+1}$$

$$\sum \dot q_s^2 = \sum_{s=1}^{m} \left[\dot y_2 \sin\frac{s\pi}{m+1} + \dot y_3 \sin\frac{2s\pi}{m+1} + \dot y_4 \sin\frac{3s\pi}{m+1} + \dots\right]^2$$

$$\frac{1}{2}\left[1-\cos\frac{2sr\pi}{m+1}\right] \qquad \sum \cos$$

$$= \dot y_2^2 \sum_{s=1}^{m} \sin^2\frac{s\pi}{m+1} + \dot y_3^2 \sum_{s=1}^{m} \sin^2\frac{2s\pi}{m+1} + \dots$$

$$+\dot y_2 \dot y_3 \sum \sin\frac{s\pi}{} \sin\frac{2s\pi}{}$$



$$x+x^2+x^3+\dots x^n = S$$

$$\frac{S}{x} = S - x^n + 1$$

$$S = Sx - x^{n+1} + x$$

$$S = \frac{x-x^{n+1}}{1-x}$$

$$e^{i\frac{s\pi}{m+1}} = \cos + i\sin$$

$$\sum{}'$$

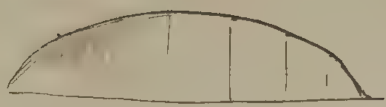
$$\left(e^{\frac{2i\pi}{m+1}}\right)^s + \left(e^{\frac{2\pi}{m+1}}\right)^s + \left(e^{\frac{3\pi}{m+1}}\right)^s + \dots \left(e^{m\frac{i\pi}{m+1}}\right)^s = \frac{e^{\frac{2\pi}{m+1}} - e^{i\pi s}}{1-e^{\frac{2i\pi}{m+1}}} = \dots \frac{\sin}{\sin\frac{s\pi}{2(m+1)}}$$

$$y_r = \sum_{s=1}^{m} q_s \sin\frac{(r-1)s\pi}{m+1}$$

$$y_{r+1} - y_r = \sum_{s=1}^{m} q_s \cos\frac{(2r-1)s\pi}{2(m+1)}\sin\frac{s\pi}{2(m+1)}$$

$$V = \frac{T}{2a}\left\{\sum_{r=1}^{m} y^2 - 2(y_1 y_2 + y_2 y_3 + \dots)\right\}$$

$s = 1$



$\sin \frac{(n-1)\pi}{n+1}$

$s = 2$

$\sin \frac{n-1}{n+1} 2\pi$

$$y_x = \sum_{s=1}^{\infty} A_s \sin s\frac{\pi x}{l} \cos n_s t$$

$$n_s = \frac{s\pi}{l}\sqrt{\frac{T}{\rho}}$$

$$\dot{y}_x = \sum A_s n_s \sin \frac{s\pi x}{l} \sin n_s t$$

$$\int \sum \left[A_s n_s \sin \frac{s\pi x}{l}\right]^2 =$$

$$Y_1 = A_1 \sin \frac{\pi}{n+1} + A_2 \sin \frac{2\pi}{n+1} + \dots$$

$$Y_2 = \dots$$

$$A_s = \frac{2}{n+1} \sum_{r=1}^{n} y_r \sin\left(r \frac{s\pi}{n+1}\right)$$

$ar = x$

$a = dx$

$$A_s = \frac{2}{l}\int_0^l y_x \sin\left(\frac{s\pi x}{l}\right) dx$$

$$\left(x+\frac{1}{x}\right)^8 = \binom{8}{0}x^8 + \binom{8}{1}x^6 + \binom{8}{2}x^4 + \binom{8}{3}x^2 + \binom{8}{4}x^0 + \binom{8}{3}\frac{1}{x^2} + \binom{8}{2}\frac{1}{x^4} + \binom{8}{1}\frac{1}{x^6} + \binom{8}{0}\frac{1}{x^8}$$

$$\left(x+\frac{1}{x}\right)^8\left(x-\frac{1}{x}\right) = \binom{8}{0}x^9 + \left[\binom{8}{1}-\binom{8}{0}\right]x^7 + \left[\binom{8}{2}-\binom{8}{1}\right]x^5 + \cdots$$

$$\frac{1}{x}\left(x+\frac{1}{x}\right)^9 = \binom{9}{0}x^8 + \binom{9}{1}x^6 + \binom{9}{2}x^4 + \binom{9}{3}x^2 + \binom{9}{4}x^0 + \binom{9}{4}\frac{1}{x^2} + \binom{9}{3}\frac{1}{x^4} + \binom{9}{2}\frac{1}{x^6} + \binom{9}{1}\frac{1}{x^8} + \binom{9}{0}\frac{1}{x^{10}}$$

$$\frac{d}{dx}\left[\frac{1}{x}\left(x+\frac{1}{x}\right)^{n}\right] = 8\binom{9}{0}x^7 + \cdots \qquad -10$$

$n^2 - (n-1)^2 = 2n - 1$

II

$$1^2(\delta_1 - \delta_2) + 2^2(\delta_2 - \delta_3) + 3^2(\delta_3 - \delta_4) + \cdots 8^2(\delta_8 - \delta_9) + 9^2\delta_9$$

$$= \delta_1 + (2^2 - 1^2)\delta_2 + (3^2 - 2^2)\delta_3 + \cdots\cdots + (9^2 - 8^2)\delta_9$$

$$= 2\delta_1 + 2.2.\delta_2 + 3.2\,\delta_3 + \cdots \qquad + 9.2.\delta_9$$

$$-\delta_1 \quad - \delta_2 \qquad - \delta_3 \qquad\qquad - \delta_9$$

$$\overline{E_m^2} = 2[\underbrace{1.\delta_1 + 2\delta_2 + 3\delta_3 + \cdots 9\delta_9}_{\frac{1}{2}\cdot 9}] - [\delta_1 + \delta_2 + \delta_3 \cdots \delta_9]$$

$$\overline{E_m^2} = m - \overline{E}_m$$

$\binom{6}{2}$

$\frac{1}{1}\,\frac{2}{1}\cdot 1$

$1 \cdot \frac{1}{2}$ 1

$1\left(\frac{1}{2}\right)^2\frac{2}{2}$ $\quad 0$ 2

$1\left(\frac{1}{2}\right)^3\frac{3}{3}$ $\quad \binom{3}{1}\left(\frac{1}{2}\right)^3\frac{1}{3}$ 3

$\binom{4}{0}\left(\frac{1}{2}\right)^4\frac{4}{4}$ $\quad \binom{4}{1}\left(\frac{1}{2}\right)^4\frac{2}{4}$ $\quad 0$ 4

$\binom{5}{0}\left(\frac{1}{2}\right)^5\frac{5}{5}$ $\quad \binom{5}{1}\left(\frac{1}{2}\right)^5\frac{3}{5}$ $\quad \binom{5}{2}\left(\frac{1}{2}\right)^5\frac{1}{5}$

$\binom{6}{0}\left(\frac{1}{2}\right)^6\frac{6}{6}$ $\quad \binom{6}{1}\left(\frac{1}{2}\right)^6\frac{4}{6}$ $\quad \binom{6}{2}\left(\frac{1}{2}\right)^6\frac{2}{6}$ $\quad 0$

$\binom{7}{0}\left(\frac{1}{2}\right)^7\frac{7}{7}$ $\quad \binom{7}{1}\left(\frac{1}{2}\right)^7\frac{5}{7}$ $\quad \binom{7}{2}\left(\frac{1}{2}\right)^7\frac{3}{7}$ $\quad \binom{7}{3}\left(\frac{1}{2}\right)^7\frac{1}{7}$

$\binom{8}{0}\left(\frac{1}{2}\right)^8\frac{8}{8}$ $\quad \binom{8}{1}\left(\frac{1}{2}\right)^8\frac{6}{8}$ $\quad \binom{8}{2}\left(\frac{1}{2}\right)^8\frac{4}{8}$ $\quad \binom{8}{3}\left(\frac{1}{2}\right)^8\frac{2}{8}$ $\quad 0$

$\binom{9}{1}\left(\frac{1}{2}\right)^9\frac{7}{9}$ $\quad \binom{9}{2}\left(\frac{1}{2}\right)^9\frac{5}{9}$ $\quad \binom{9}{3}\left(\frac{1}{2}\right)^9\frac{3}{9}$ $\quad \binom{9}{4}\left(\frac{1}{2}\right)^9\frac{1}{9}$

$\binom{10}{1}\left(\frac{1}{2}\right)^{10}\frac{8}{10}$ $\quad \binom{10}{2}\left(\frac{1}{2}\right)^{10}\frac{6}{10}$ $\quad \binom{10}{3}\left(\frac{1}{2}\right)^{10}\frac{4}{10}$ $\quad \binom{10}{4}\left(\frac{1}{2}\right)^{10}\frac{2}{10}$ $\quad 0$

$$\frac{1}{2^n}\left\{\sin n\varphi + \frac{n-2}{n}\binom{n}{1}\sin(n-2)\varphi + \frac{n-4}{n}\binom{n}{2}\sin(n-4)\varphi + \cdots \quad n.\frac{\sin\varphi}{\sin\varphi}\right\} = \frac{1}{2}\sin\varphi\cos^{n-1}\varphi$$

$$\sum_{n=1}^{m}\sum_{\mu=n}^{m} a_{n\mu}\sin n\varphi = \frac{1}{2}\sin\varphi\underbrace{\sum_{n=1}^{m}\cos^{n-1}\varphi}$$

$$1+\cos\varphi+\cos^2\varphi+\cdots\cos^{n-1}\varphi = \frac{1-\cos^m\varphi}{1-\cos\varphi}$$

$$\int_0^{\frac{\pi}{2}} \sin^{2a} x \cos 2bx \, dx = 0 \qquad a < b$$

$$\int_0^{\frac{\pi}{2}} \sin^{2a} x \cos 2bx \, dx = \frac{(-1)^b}{2^{2a+1}} \pi \binom{2a}{a-b} \qquad a > b$$

für $b = 0$:

$$\int_0^{\frac{\pi}{2}} \sin^{2a} x \, dx = \frac{\pi}{2^{2a+1}} \binom{2a}{a} = \frac{\pi}{2^{2a+1}} \frac{2a(2a-1)(2a-2)\cdots(a+1)}{1.\,2.\,3.\,\cdots\,a} = \frac{\pi}{2^{2a+1}} \frac{2a!}{(a!)^2}$$

$a = k-1$

$$\sum A = \sum_{k=1}^{k=m} \binom{m}{k} (-1)^{k-1} \frac{2^{k-1}}{2^{2k+1}} \cdot \frac{(2k-2)!}{((k-1)!)^2}$$

$$+ \sum \binom{m}{k} (-1)^{k-1} \frac{2^{k-1}(-1)}{2^{2k+1}} \binom{2k-2}{k-2}$$

$$= \sum \binom{m}{k} (-1)^{k-1} \frac{1}{2^{k+2}} \left[\binom{2k-2}{k-1} - \binom{2k-2}{k-2} \right]$$

$$\binom{2a}{a} - \binom{2a}{a-1} =$$

$$\frac{2a(2a-1)\cdots(a+1)}{a!} - \frac{2a(2a-1)\cdots(a+2)}{a-1!}$$

$$= \frac{2a(2a-1)\cdots[(a+1)-a]}{a!}$$

$$= \frac{2a(2a-1)\cdots(a+2)}{a!} = \binom{2a}{a-1}\frac{1}{a}$$

$$= \binom{2a}{a}\frac{1}{a+1}$$

$$= \sum_{k=1}^{k=m} (-1)^{k-1} \binom{m}{k} \frac{1}{2^{k+2}} \binom{2k-2}{k-1} \frac{1}{k}$$

10 9 8 7 6 5 4 3 2 1 0

$\cancel{\frac{1}{n 2^n}}$ $\cancel{\Sigma}$

1 6 15 20

~~116~~

1 7 21 35

1 8 28 56 70

1 9 36 84 126

1 10 45 120 210 252

$$n + (n-2)\cdot\frac{n(n-1)}{1.2} + (n-4)\,\frac{n(n-1)(n-2)(n-3)}{1.2.3.4} + (n-6)\,\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{1.2.3.4.5.6}$$

$$A_n = \sum_{\mu=n}^{m} a_{n\mu}$$

$$\sin\alpha \cos\beta = \frac{\sin(\alpha+\beta) + \sin(\alpha-\beta)}{2}$$

$$\sum_{n=1}^{m} A_n \sin n\varphi = \frac{1}{2}\sin\varphi \; \frac{1-\cos^m\varphi}{1-\cos\varphi}$$

$$\sin m\psi \cos\psi = \frac{\sin(m+1)\psi}{2} + \frac{\sin(m-1)\psi}{2}$$

$$A_n = \frac{2}{\pi}\int_0^{\pi} \frac{1}{2}\sin\varphi \sin n\varphi \, \frac{[1-\cos^m\varphi]}{1-\cos\varphi}\, d\varphi$$

$$\sin\alpha \sin\beta = \frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{2}$$

$$\sum_{n=1}^{m} \sin n\varphi = \sin\varphi + \sin\varphi + \cdots \sin m\varphi = \frac{\sin\frac{m\varphi}{2}\sin\frac{m+1}{2}\varphi}{\sin\frac{\varphi}{2}}$$

$$\sum_{n=1}^{m} A_n = \frac{1}{\pi}\int_0^{\pi} \frac{\sin\varphi \sin\frac{m\varphi}{2}\sin\frac{(m+1)\varphi}{2}\,[1-\cos^m\varphi]}{\sin\frac{\varphi}{2}\;[1-\cos\varphi]}\, d\varphi =$$

$$= \frac{1}{\pi}\int_0^{\pi} \frac{\cos\frac{\varphi}{2}\sin\frac{m\varphi}{2}\sin\frac{(m+1)\varphi}{2}}{\sin^2\frac{\varphi}{2}}\,[1-\cos^m\varphi]\, d\varphi$$

$$\frac{\varphi}{2} = \psi$$

$$d\varphi = 2\,d\psi$$

$$= \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{\cos\psi \sin m\psi \sin(m+1)\psi}{\sin^2\psi}\,[1-\cos^m 2\psi]\, d\psi$$

$$= \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \left[\frac{\sin^2(m+1)\psi}{2\sin^2\psi} + \frac{\sin(m+1)\psi\,\sin(m-1)\psi}{2\sin^2\psi}\right][1-\cos^m 2\psi]\, d\psi$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \underbrace{\frac{1-\cos^m 2\psi}{\sin^2\psi}}\left\{\frac{1-\cos 2(m+1)\psi}{2} + \frac{\cos 2\psi - \cos 2m\psi}{2}\right\} d\psi$$

$$\overbrace{\frac{1-(1-2\sin^2\psi)^m}{\sin^2\psi}} = \left\{\binom{m}{1}\cdot 2 \cancel{\quad} - \binom{m}{2}\cdot 2^2\sin^2\psi + \binom{m}{3}\cdot 2^3\sin^4\psi - \binom{m}{4}\cdot 2^4\sin^6\psi + \cdots\right\}$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left\{\binom{m}{1} - \binom{m}{2}2^2\sin^2\psi + \binom{m}{3}2^2\sin^4\psi - \binom{m}{4}2^3\sin^6\psi + \cdots + (-1)^{\cdot}\binom{m}{m}2^{m-1}\sin^{2m-2}\psi\right\}\left\{1+\cos 2\psi - \cos 2m\psi - \cos 2(m+1)\psi\right\} d\psi$$

$$\frac{1}{1.2.3}\left\{ \frac{\cancel{7}\cdot 6.5}{2^7}\cdot 1 + \frac{\cancel{8}.7.6}{2^8}\cdot 2 + \frac{\cancel{9}\,8.7}{2^9}\cdot 3 + \frac{\cancel{10}.9.8}{2^{10}}\cdot 4 \right.$$

$$\frac{1}{1.2.3.4}\left\{ \frac{\cancel{9}\,8.7.6}{2^9}\cdot 1 + \frac{\cancel{10}.9.8.7}{2^{10}}\cdot 2 + \frac{\cancel{11}\,10.9.8}{2^{11}}\cdot 3 + \frac{\cancel{12}.11.10.9}{2^{12}}\cdot 4 + 1 \right.$$

$$\frac{d}{dy}\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}\left\{ \left(\frac{x}{2}\right)^8 y + \left(\frac{x}{2}\right)^9 y^2 + \left(\frac{x}{2}\right)^{10} y^3 + \left(\frac{x}{2}\right)^{11} y^4 \right.$$

$$\int \frac{\cos px}{\cos^2 x}\,dx = 2^{p-1}\pi \qquad \int \frac{\cos 2ax \cos px}{\cos^p x}\,dx = (-1)^a\, 2^{p-\cancel{2}}\pi \quad b$$

[illegible]

Ti – [illegible] – [illegible] – x – Bi

?1 95 [illegible] [illegible] 210

[illegible]

[illegible]

[illegible]

Mg [illegible] [illegible] x [illegible]

24 [illegible] [illegible] 156 200

x – Cu – Ag – x – [illegible]

[illegible] 63 [illegible] 153 197

[illegible] – [illegible] – [illegible] – x – Pb

27 72 117 [illegible] 206

x – [illegible] – [illegible] – x – [illegible]

8 52 76 [illegible] 184

S – Se Te

32 [illegible] 126

Ca – Sr – Ba

40 87 [illegible]

[illegible]

[illegible] [illegible] [illegible] x [illegible]

[illegible] 70 [illegible] 158 [illegible]

[illegible] [illegible] [illegible] [illegible] [illegible]

[illegible] [illegible] [illegible] [illegible] [illegible]

[illegible] – Na – K – Rb – Cs

7 23 39 85 133

[illegible]

18.0 [illegible] [illegible] [illegible] [illegible]

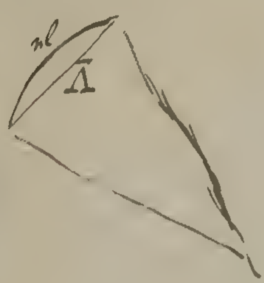
[illegible] 216 [illegible] [illegible]

V Nb x Ta

51 94 13? [illegible]

Wenn das als Kreisbogen aufgefasst wird.

Formel (11):



$$\frac{\bar{\Lambda}}{nl} = 1 - \frac{n\delta}{6} = \frac{2a \sin\frac{\varphi}{2}}{a\varphi} = 1 - \frac{\left(\frac{\varphi}{2}\right)^2}{2.3}$$

$$\left(\frac{\varphi}{2}\right)^2 = n\delta$$

$$\varphi = 2\delta\sqrt{n}$$

Vergl. Rieger Pr. R. S. 83 p. 499

Rayleigh Theory of Sound 2 ed. 1894 p. 39

$$\Lambda = C\sqrt{\frac{2M}{S}}$$

$$M\frac{d^2x}{dt^2} = -S\frac{dx}{dt} = \quad \cancel{S = \frac{2\pi}{3}R^2} \; mNc$$

$$V = Ce^{-\frac{t}{\frac{M}{S}}}$$

$$= C\sqrt{\frac{2M}{\frac{2}{3}mn}} = c\sqrt{\frac{2m}{\frac{2mn}{3}}} = c\sqrt{\frac{3}{n}}$$

$$= NR^2\pi c \,.\, \frac{2m}{3M}$$

$$S = \frac{2\pi}{3}R^2\rho c = \frac{2\pi}{3}R^2Nmc$$

$$S = 6\pi R\mu$$

$$S = \frac{2\pi mcR^2N}{3M}$$

$$\frac{8}{3}\,4R^2N\frac{\sqrt{\pi Mm}}{m+M} \neq \frac{32}{3}R^2N\sqrt{\frac{\pi m}{M}}$$

zuputhie [illegible]

[illegible]

[illegible] v $\frac{c}{n} = \lambda$

~~[illegible]~~

$\sigma = a \sin 2\pi n\left(t - \frac{x}{c}\right) = a \sin 2\pi\left(nt - \frac{x}{\lambda}\right)$ [illegible]

$\sigma' = a \sin 2\pi n \frac{c}{c-v}\left(t - \frac{x}{c}\right) = a \sin 2\pi\left(\frac{[illegible]\,t}{c-v} - \frac{x}{[illegible]}\right)$

[illegible] $\lambda' = \frac{c-v}{n}$

[illegible]

$\sigma' = a \sin 2\pi n\left(\frac{ct}{c-v} - \frac{x}{c-v}\right)$

[illegible]

$\sigma' = a \sin 2\pi n\left(t - \frac{x}{c-v}\right)$

[illegible]	[illegible]	[illegible]	[illegible]
M A	E	+	
E	M A		
M E	A	+	+
A	M E		
F A	H		
H	F A		+

$[illegible] = M$

$[illegible] = N$

$x^2 + [illegible] x_1 = M$

$x_1 = -\frac{M}{2} \pm \sqrt{[illegible] - N}$

$\frac{d x_1}{dt} = -\frac{1}{2}\frac{dM}{dt} \pm \frac{1}{\sqrt{\frac{M^2}{4} - N}}\left[\frac{M}{4}\frac{dM}{dt} + \frac{dN}{dt}\right]$

$= 0$

$[illegible] = 0$

$\sqrt{M [illegible]} = \pm [illegible]$

$[illegible] = \frac{M^2}{4} + M q - [illegible]$

$q = \frac{[illegible] - v_0 [illegible]}{[illegible]} = v_0$

$q = v_0 [illegible] \left[\frac{[illegible]}{T_0 [illegible]} n - \frac{\sigma}{n^2}\right]$

$\underbrace{[illegible]}_{C}$

$\frac{[illegible] \frac{T}{T_0} - [illegible] + [illegible] + C]}{[illegible]} =$

$= \frac{[illegible]}{[illegible]} [illegible] n - \frac{\sigma}{n^2}]$

$+ [illegible] q [illegible]^2 \frac{[illegible]}{v_0}$

A) $c \frac{t}{[illegible]} - [illegible] + [illegible] n [illegible] \left[[illegible] - [illegible]\right]$

$= t [illegible] - \frac{M [illegible]}{[illegible]} [illegible]$

$= t [illegible] \frac{n}{[illegible]}]$

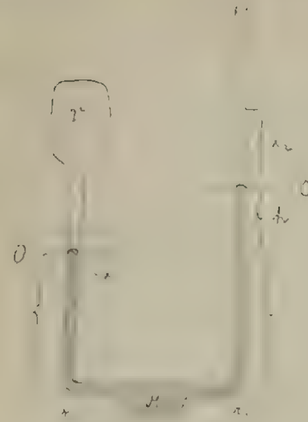
I $[illegible] = [illegible]$ $q = 0$

II $[illegible] - d = 0$

B) $[illegible]$

$[illegible] \left[[illegible] + 1 \right] = 0$

$N=0$



$p\,v = R\,T$

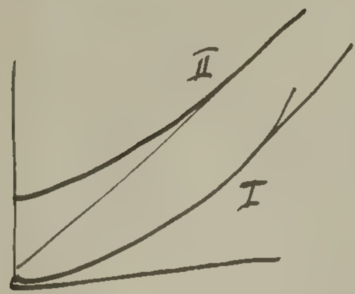
$p_0 v_0 = R\,T_0$

$p_0 = 760 + h$

$h = 760 - $

$A_y - A_y$

$$\cancel{\frac{h\nu}{\frac{h\nu}{kT}+\left(\frac{h\nu}{kT}\right)^2}} = \cancel{kT\left(1+\frac{h\nu}{2kT}\right)}$$



I). $E = \dfrac{h\nu}{e^{\frac{h\nu}{kT}}-1}$

II). $E = \uparrow + \frac{h\nu}{2}$

I). $\lim_{T=\infty} E = kT - \frac{h\nu}{2}$

II). $\qquad = kT$

$$\sigma = \frac{C}{\lambda^5}\,\frac{1}{e^{\frac{hc}{k\lambda\theta}}-1}$$

Rayleigh $\quad \dfrac{C\,k\,\theta}{\lambda}$

$$\int_0^y e^{-\beta y^2} dy \left[\int_0^y e^{-\alpha x^2} dx\right]^{n-1} = \left[\int_0^y e^{-\alpha x^2} dx\right]^{n-1} \int_0^y e^{-\beta y^2} dy - \int dy\,(n-1)\, e^{-\alpha y^2} \left[\int_0^y e^{-\alpha x^2} dx\right]^{n-2} \cdot \int_0^y e^{-\beta y^2} dy$$

$$= \int_0^{\infty} e^{-\alpha y^2} \cdot e^{-(\beta-\alpha) y^2} dy \Big| \ldots = \frac{1}{n}\left[\frac{1}{2}\sqrt{\frac{\pi}{\alpha}}\right]^n \cdot e^{-(\beta-\alpha) y^2} \Big|_0^{\infty} + \int 2(\ \ )-\alpha)\, y\, e^{-(\beta-\alpha) y^2} \left[\int_0^y\ \ \right]^n dy$$

$$2\nu P + 2\nu^2 P + \nu^2 P^2 (2 - 1 + 2 + 2 - 1)$$

$$+ \nu^2 P^3 \left(\frac{1}{2} + \frac{1}{2}\right.$$

$$m^2 \binom{n}{m} = \frac{n(n-1)(n-2)\cdots(n-m+1)}{m!} m^2 =$$

$(n-m)^2 \cdot \cdots \quad (m)^2$

$n - m + 1 \ldots$

$2n - m$

$-m$

$n + i = -m$

$i = n + m$

$$(1)^2\left[\frac{b_1}{1!}\frac{b}{0!} + \frac{b^2}{2!}\frac{b^1}{1!} + \frac{b^3}{3!}\frac{b^2}{2!} + \frac{b^4}{4!}\frac{b^3}{3!} + \frac{b^5}{5!}\frac{b^4}{4!}\right.$$

$$0\left[\frac{b^0}{0!} + \frac{b}{1!}\frac{b^1}{1!} + \frac{b^2}{2!}\frac{b^2}{2!} + \frac{b^3}{3!}\frac{b^3}{3!} + \right.$$

$$+(1)^2\left[\frac{b^1}{1!}\frac{b^0}{0!} + \frac{b^2}{2!}\frac{b^1}{1!} + \frac{b^3}{3!}\frac{b^2}{2!} + \frac{b^4}{4!}\frac{b^3}{3!}\right.$$

$$+(2)^2\left[\frac{b^2}{2!}\frac{b^0}{0!} + \frac{b^3}{3!}\frac{b^1}{1!} + \frac{b^4}{4!}\frac{b^2}{2!} + \cdots\right.$$

$$\cancel{\sum_{m=1}^{\infty} m \frac{b^m}{m!} = b e^b - 1}$$

$$+(3)^2\left[\frac{b^3}{3!}\frac{b^0}{0!} + \frac{b^4}{4!}\frac{b^1}{1!} + \frac{b^5}{5!}\frac{b^2}{2!} + \cdots\right.$$

$$= \sum_{m=0}^{\infty} \frac{b^m}{m!}\left[0\cdot\frac{b^m}{m!} + (1)^2\frac{b^{m+1}}{m+1!} + (2)^2\frac{b^{m+2}}{m+2!} + \right.$$

$$= \sum_{m=0}^{\infty} \frac{b^m}{m!}\left[m^2\frac{b^0}{0!} + (m-1)^2\frac{b^1}{1!} + (m-2)^2\frac{b^2}{2!} + (m-3)^2\frac{b^3}{3!} + \cdots + (2)^2\frac{b^{m-2}}{m-2!} + (1)^2\frac{b^{m-1}}{m-1!} + 0 b^m\right]$$

$$+ (1)^2\frac{b^m}{m!} + (2)^2\frac{b^{m+1}}{m+1!} + \cdots$$

$$+ \sum_{m=1}^{\infty} \frac{b^m}{m!}\left[m^2\frac{b^0}{0!} + (m+1)\frac{b^1}{1!} + (m+2)^2\frac{b^2}{2!} + (m+3)^2\frac{b^3}{3!} + \cdots\right]$$

$$x^m e^{\alpha x} = x^m + \alpha\frac{x^{m+1}}{1!} + \alpha^2\frac{x^{k+2}}{2!} + \alpha^3\frac{x^{k+3}}{3!} + \cdots$$

$$x\frac{d}{dx}(x^m e^{\alpha x}) = m x^{m} + (m+1)\frac{\alpha x^{m+1}}{1!} + \frac{(m+2)\alpha^2 x^{m+2}}{2!} + \cdots$$

$$\frac{d}{dx}\left[x\frac{d}{dx}(x^m e^{\alpha x})\right]_{x=1} = m^2 + (m+1)^2\frac{\alpha}{1!} + (m+2)^2\frac{\alpha^2}{2!} + \cdots = \frac{d}{dx}\left[x\cdot\left(m x^{m-1}e^{\alpha x} + \alpha x^m e^{\alpha x}\right)\right]$$

$$= m^2 e^{\alpha} + \alpha(m+1)e^{\alpha}$$

$$e^b\left[b^2 - 2bm + b + m^2 + m^2 + b(m+1)\right]$$

$$= e^b\left[b^2 - bm + 2b + 2m^2\right] \qquad e^b\left[(b^2+2b)e^b - b\cdot b e^b + 2(b+b^2)e^b\right]$$

$$x_0^2\left(1 + e^{-2\beta t}\right) + \xi^2\left(1 - e^{-2\beta t}\right) - 2x_0^2 e^{-\beta t}$$

$$= x_0^2\left(1 - e^{-\beta t}\right)^2 + \xi^2\left(1 - e^{-2\beta t}\right)$$

1241 1562 19368 98677 02119 00945 73799
2803 0097 86082 86082 86082 86082 86082
2900 1050 05450 84759 88201 87027 59881
1850 1022 2900 1850 0828
0828 0547 71337 07040 07621 07418 07970
0281 1241 2803
2375 ~~1594~~ 2138 1108 0431
2873

0901 1474 16850 69723 86629 01284 83059
2375 0498 86082 86082 86082 86082 86082
2873 0735 02932 55805 72711 87366 69141
2138 1030 ~~10691~~ 07614 2873 2138 1108
1108 0677 1070 2375 05335 07476 04914
0431 0901 2340 1390 0617
1971 2736

2356 . 164
1413
97
386

354
212
14
85

25 .

164
76
2
18

171
34
15
22

3216
643
27
20
414

278
556
250
359

15 . 129
645
1935

575
115 .
52
742

32 . 24
64
128
77

129 . 4
516

69
138
193

29 . 69
174
261
20

86
43
13

29 . 32

21 . 32
63
4

83 . 69
498
747
572

325
32
3
36

32
32
365
36
4
405

205
20
2
23

77
~~74~~
854

22
2

$P = \frac{2\cdot25}{3\cdot10} =$

35218
49136
86082
19033
05115

$P = 0.7258$

$P^2 = 0.5268..$

$Pv = 11250$

$$P(n-v)^2 - P(n-v) - P^2 n + 2Pn$$

$$\Delta_n^2 = \cancel{P(n-v)\left[1+P(n-v)\right]} - nP^2$$

$$= P^2(n-v)^2 - (n-v)P - nP^2 + 2nP$$

$$= \left[P(n-v) - \tfrac{1}{2}\right]^2 - \tfrac{1}{4} - n(P-1)^2 + n$$

$$= \left[P(n-v) - \tfrac{1}{2}\right]^2 + n\underbrace{\left[1-(P-1)^2\right]}_{0.92481} - \tfrac{1}{4}$$

·05115
09691
14806

-0.14062

0.2742
0.87614
0.07519

1.6250
21085
42170

+2.6406
− 25
− 2.3906
+

− 1.6250
+ 7258
0.8992

95386
90772
+0.80858
92481
1.7334
− 25
1.4834

− 1.6250
+ 1.4516
0.1734
23905
47810

0.03007
184962
1.8797
− 25
1.6297

27774
1.6250
0.5524
74225
48450

0.30514
2.77493
3.0796
− 25
2.8296

2.9032
1.6250
1.2782
10660
21320

1.6338
3.6992
53330
− 25
5.083

3.6290
1.6250
2.0040
40160
41240
8.640
− 25
8.390

0.43429. 1125
4343
869
217
0.48858
0.51142
0.3247

51142	51142	51142	51142	51142
05115	10230	15345	20460	25575
56257	61372	66487	71602	76717

3652   4109   4622   5200

2054   07703   1300

02107

07918
68799
·004875

51142
86082
37224
86082
23306
09388

4875
914
3961. 7258

95470
81552

59780
86082
45862
4875
− 002875
0020

51142
30690
81832
− 85733
96099
·0009141

$\frac{\nu^n e^{-\nu}}{n!}$

$\nu = 1.55$

·19033	2·04118	·1704
	·23151	
38066	42184	2641
57099	61217	4094
76132	80250	6346
		1058
95165	99283	9837
14198	18316	1525
33231	37349	2363

·128

1099
1704
1321
68·2
26·45
8·20
1·81
0·35

108·7
168·4
130·6
67·4
26·15
8·05
1·79

164
164
8
1812

43729
21714
21715
·67315
·32685 −1
2·71433
2·04118

$\frac{6}{5\cdot18} = 116\ \%$

1099
12
108·7

1321
15
130·6

170·4
17
2
168·4

1247
260
23
35·3

3652
272
236
39·48

68·2
68
7

820
9

2645

2194
54
4
22·4

7609
761
8·37

218
11
7
2·36

488.109
4392
0·532

181

67·4
1348
265
8·35

1348
539
4
18·9

2356
1414
188
9
39·67

3541
2125
283
14
59·6

2492
1495
195
10
41·9

1684
168
33
3
18·88

3685
2211
295
15
62·06

168
8
16

1348
6066
19·5

674
539
34
12·47

5392
135
54
55·8

171
513
10
22·3

3216
965
19
42·0

278
834
16
36·3

1306
653
19·6

575
1725
34
7·51

1306
914
2·22

0·5

1·9

0·3652
0·3247
0·0405 . 7258
29032
363
0·029395
3247
0·3541
·17102

3652
20545
·15975 . 7258
16 . 725
4350
− ·11600
+ 3652
0·2492

20545
07703
·12842
10864
86082
96946
−0·09321
+ 20545
0·11224

0·7703
2167
·05536
74320
86082
60402
−0·04018
+ 07703
0·03685

2167
487
·01680
22531
86082
08613
0·02167
−0·01219
+0·00948

2356
3541
2492
1122
03685
00948
200

1185
1049
1370
07535
02737
00748

07072
86082
93454
87708
86082
73790

·02078
86082
88160
43727
86082
29809

13672
86082
99754
1122
·0547
·0575

0·08601
2356
·3216
3541
0·07614
·2780
03685
·019865
·01698

2492
·099935
·1498
51142

87390
86082
73472

948
·00543
·00405

1710
3216
2780
1498
0575
0170

1506
0436
1282
0923
0405

17782
86082
03864
+ ·1093
1710
·2803

63949
86082
50031
3216
03165
2900

10789
86082
96871
2780
09305
1850

96520
86082
82602
1498
0674
0828

60740
86082
46828
0575
0294
0281

$$(1+x)^{1/2} = 1 + \frac{x}{2} - \frac{x^2}{8}$$

$$P^2 + \frac{n+\nu}{(n-\nu)^2 - n} P = \frac{1}{(n-\nu)^2 - n}$$

$$P = -\frac{n+\nu}{2\left[(n-\nu)^2-n\right]} \pm \sqrt{\frac{1}{\left[(n-\nu)^2-n\right]} + \frac{(n+\nu)^2}{4\left[\quad\right]^2}}$$

$$= \frac{-(n+\nu) \pm \sqrt{4\left[(n-\nu)^2-n\right] + (n+\nu)^2}}{2\left[(n-\nu)^2-n\right]}$$

Falls $n \ll (n-\nu)^2$

$1+\delta \ll \nu\delta^2$

$1 \gg \delta^2 \gg \frac{1}{\nu}$

$$P = \frac{-(n+\nu) \pm \sqrt{4(n-\nu)^2 + (n+\nu)^2}}{2(n-\nu)^2} = \frac{-(2+\delta) + \sqrt{4\delta^2 + (2+\delta)^2}}{2\nu\delta^2}$$

$$= \frac{-(2+\delta) + \sqrt{4+4\delta+5\delta^2}}{2\nu\delta^2}$$

$$= \frac{-(2+\delta) + 2\left[1+\delta+\frac{5}{4}\delta^2\right]^{1/2}}{2\nu\delta^2}$$

$$= \frac{\cancel{1} + \cancel{\frac{\delta}{2}} + \frac{5}{8}\delta^2 - \frac{\delta^2}{8} - \cancel{1} - \cancel{\frac{\delta}{2}}}{\nu\delta^2} = \frac{1}{2\nu}$$

$$P = \frac{-(n+\nu) + (n+\nu)\left[1 + \frac{4\left[(n-\nu)^2-n\right]}{(n+\nu)^2}\right]^{1/2}}{2\left[(n-\nu)^2-n\right]} = \frac{(n+\nu)\left\{\frac{2\left[(n-\nu)^2-n\right]}{(n+\nu)^2} - 2\frac{\left[(n-\nu)^2-n\right]^2}{(n+\nu)^4}\right\}}{2\left[(n-\nu)^2-n\right]}$$

$$= \frac{1}{n+\nu} - \frac{(n-\nu)^2-n}{(n+\nu)^3} \;\cdots\cdots$$

$$= \frac{1}{n+\nu}\left\{1 - \frac{(n-\nu)^2-n}{(n+\nu)^2} \cdots\cdots\right\}$$

Falls $\frac{(n-\nu)^2-n}{(n+\nu)^2} \ll 1$ immer erfüllt!

$\cancel{\nu\delta^2 - (1+\delta) \ll (2+\delta)^2}$ $\cancel{\ll 1}$ $\cancel{\nu\delta^2 - 1 - \delta \ll \nu\delta}$

$\cancel{n^2} \cancel{+\nu^2} - 2n\nu - n \ll \cancel{n^2} \cancel{+\nu^2} + 2n\nu$

aber Entwicklung erfordert eigentlich

$\cancel{+ 4\nu} + 4\nu\delta \quad \ll \frac{1}{4}$

also

$\nu\delta^2 - 1 - \delta \ll \frac{\nu\delta^2}{4} + \nu + \nu\delta$

$\frac{3}{4}\nu\delta^2 - 1 - \delta \ll \nu + \nu\delta$

$-1-\delta \ll \nu + \nu\delta - \frac{3}{4}\nu\delta^2$

Jedenfalls gültig falls $\delta < 1$

$\frac{26.15}{2.35}$ $\quad$ $\frac{237 \cdot 262}{474}$ $\quad$ $\frac{287 \cdot 262}{574}$ $\quad$ $\frac{214 \cdot 261}{428}$ $\quad$ 2615

$\quad$ 142 $\quad$ 172 $\quad$ 128 $\quad$ 2615

$\quad$ 5 $\quad$ 6 $\quad$ 2 $\quad$ 21

$\quad$ 6.21 $\quad$ 7.52 $\quad$ 5.58 $\quad$ 2.8975 $\quad$ 1.1

0.5 $\quad$ 1.6 $\quad$ 2.2 $\quad$ 1.9 $\quad$ 1.1 $\quad$ 0.4

---

1) $\left[1+\frac{b^2}{1!}+\frac{b^4}{12\cdot(2,3)}+\frac{b^6}{3!\,2.3.4}+\cdots\right]\frac{b}{1}$

$+\left[1+\frac{b^2}{1!}+\right.$

$+\cancel{\frac{b^2}{1!}}$ 1). $1\left\{\frac{b}{1!}+\frac{b^3}{1!\,2!}+\frac{b^5}{2!\,3!}+\frac{b^7}{3!\,4!}+\cdots\right\}+$

2). $+4\left\{\frac{b^2}{2!}+\frac{b^4}{1!\,3!}+\frac{b^6}{2!\,4!}+\frac{b^8}{3!\,5!}+\cdots\right\};+$

$+9\left\{\frac{b^3}{3!}+\frac{b^5}{1!\,4!}+\frac{b^7}{2!\,5!}+\right.$

$+16\left\{\frac{b^4}{4!}+\frac{b^6}{1!\,5!}+\cdots\right.$

$\frac{(b)^n}{n!}\left[1+\frac{b^2}{1!}+\frac{b^4}{2!(n+1)(n+2)}\right.$

$=\sum_{k=-n}^{\infty}k^2\sum_{n=0}^{\infty}\frac{b^{k+n}}{k+n!}\,\frac{b^n}{m!}=\sum'\ \sum'\frac{b^m}{m!}\ b^n$

$=(-n)^2\sum_{m=0}^{\infty}\left(\frac{b^{m+n}}{m+n!}\,\frac{b^m}{m!}\right)+(-n+1)^2\sum\left(\frac{b^{m+n-1}}{m+n-1!}\,\frac{b^m}{m!}\right)+\cdots\ 0\cdot\sum\frac{b^m}{m!}\,\frac{b^m}{m!}+\cdots$

$=\frac{b^0}{0!}\left[(-n)^2\sum_{m=0}^{\infty}\frac{b^{m+n}}{m+n!}+(-n+1)\sum\frac{b^{m+n-1}}{m+n-1!}+\cdots\right.$

$+\frac{b^1}{1!}\left[(-n)^2\sum\right.$

$$+(2)^2\left[\frac{b^2}{2!}\frac{b^0}{0!}+\frac{b^3}{3!}\frac{b^1}{1!}+\frac{b^4}{4!}\frac{b^2}{2!}+\frac{b^5}{5!}\frac{b^3}{3!}+\frac{b^6}{6!}\frac{b^4}{4!}+\cdots\right.$$

$$+1\left[\frac{b^1}{1!}\frac{b^0}{0!}+\frac{b^2}{2!}\frac{b^1}{1!}+\frac{b^3}{3!}\frac{b^2}{2!}+\frac{b^4}{4!}\frac{b^3}{3!}+\frac{b^5}{5!}\frac{b^4}{4!}+\cdots\right.$$

$$0\left[\frac{b^0}{0!}\frac{b^0}{0!}+\frac{b^1}{1!}\frac{b^1}{1!}+\frac{b^2}{2!}\frac{b^2}{2!}+\frac{b^3}{3!}\frac{b^3}{3!}+\frac{b^4}{4!}\frac{b^4}{4!}+\cdots\right.$$

$$+1\left[\frac{b^1}{1!}\frac{b^0}{0!}+\frac{b^2}{2!}\frac{b^1}{1!}+\frac{b^3}{3!}\frac{b^2}{2!}+\frac{b^4}{4!}\frac{b^3}{3!}+\frac{b^5}{5!}\frac{b^4}{4!}+\cdots\right.$$

$$+(2)^2\left[\frac{b^2}{2!}\frac{b^0}{0!}+\frac{b^3}{3!}\frac{b^1}{1!}+\frac{b^4}{4!}\frac{b^2}{2!}+\right.$$

$$+(3)^2\left[\frac{b^3}{3!}\frac{b^0}{0!}+\frac{b^4}{4!}\frac{b^1}{1!}+\right.$$

$$+(4)^2\left[\frac{b^4}{4!}\frac{b^0}{0!}+\right.$$

$$v^2 = 2\frac{p}{\rho}$$

$$\frac{v^2\rho}{2} - p$$

$$\sigma = \frac{10^6}{9.10^{11}}$$

$$\eta = 0{\cdot}02$$

$$K\Delta p = \frac{4}{300}$$

$$E = \frac{4}{4\pi} \cdot \frac{P}{9.10^5} \; \frac{1}{0{\cdot}02}$$

pro Atmosph.: $P = 10^6$

$$E = \frac{1}{9\pi} \cdot 50 \,.\, 10 = 20 \text{ Volt}$$

$$\Delta P = \frac{2\,E\,K\Delta\varphi}{R^2\pi} = \frac{2\,(K\Delta p)^2}{4\pi^2}\;\frac{P\sigma}{\eta}\;\frac{1}{R^2}$$

$R = 0{\cdot}1$ mm

$$\frac{\Delta P}{P} = \left(\frac{K\Delta\varphi}{\pi}\right)^2 \frac{1}{2}\;\frac{\sigma}{\eta}\;\frac{1}{R^2} = \frac{1}{2}\left(\frac{4}{1000}\right)^2 \frac{10^{-5}}{9}\;\frac{1}{0{\cdot}02}\;\frac{1}{10^{-4}}$$

$$= \frac{16}{2.9}\;\frac{50.\,10^{-5}}{10^{6-4}} = 5\cdot\frac{10^{-4}}{10^{2}} = 5.10^{-6}\,!$$

Hella i Imesa dają wyniki najzupełniej zgodne i tem samem umacniają podstawy poprzednich obliczeń!

$\cos\varepsilon = \cos\alpha_1\cos\alpha_2 + \cos\beta_1\cos\beta_2 + \cos\gamma_1\cos\gamma_2$ $\qquad = (\cos\alpha_1\cos\beta_2 - \cos\alpha_2\cos\beta_1)^2 + (\quad)^2 + (\quad)^2$

$$\sin\varepsilon = \sqrt{1-(\quad)^2} = \cancel{\cos^2\alpha_1\cos^2\alpha_2} + \cos^2\alpha_1\cos^2\beta_2 + \cos^2\alpha_1\cos^2\gamma_2 + \cos^2\beta_1\cos^2\alpha_2 + \cos^2\beta_1\cos^2\beta_2 + \cos^2\beta_1\cos^2\gamma_2 + \cos^2\gamma_1\cos^2\alpha \ldots - \left\{\begin{array}{l} \cancel{\cos^2\alpha_1\cos^2\alpha_2} + \cos^2\beta_1\cos^2\beta_2 + \ldots \\ 2\cos\alpha_1\cos\beta_1\cos\alpha_2\cos\beta_2 - \end{array}\right.$$

$\cos\frac{\varepsilon}{2} =$

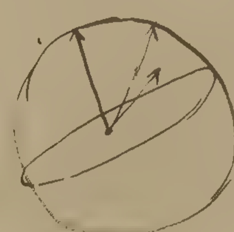
$\cos\alpha_1\cos\lambda + \cos\beta_1\cos\mu + \cos\gamma_1\cos\nu = 0$ $\qquad (\cos\alpha_1\cos\gamma_2 - \cos\alpha_2\cos\gamma_1)\cos\lambda + (\cos\beta_1\cos\gamma_2 - \cos\beta_2\cos\gamma_1)\cos\mu = 0$

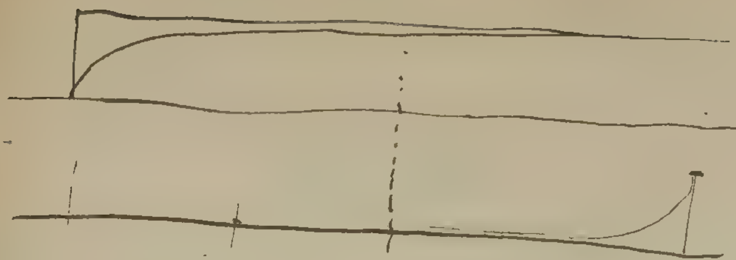
$\cos\alpha_2\cos\lambda + \cos\beta_2\cos\mu + \cos\gamma_2\cos\nu = 0$

$\cos^2\lambda + \cos^2\mu + \cos^2\nu = 1$

$$\frac{\cos\lambda}{\cos\beta_1\cos\gamma_2 - \cos\beta_2\cos\gamma_1} = \frac{\cos\mu}{\cos\gamma_1\cos\alpha_2 - \cos\gamma_2\cos\alpha_1} = \frac{\cos\nu}{\quad} = K$$

$$\cos\lambda = \frac{\cos\beta_1\cos\gamma_2 - \cos\beta_2\cos\gamma_1}{\sqrt{\quad}}$$





$$\sin\left(n\pi - \frac{n\pi\xi}{c}\right)$$
$$= (-1)^{n+1}\sin\frac{n\pi\xi}{c}$$

$$u = \gamma\left\{\frac{x}{c} + \frac{2}{\pi}\sum\frac{(-1)^n}{n}\, e^{-a^2\left(\frac{n\pi}{c}\right)^2 t}\,\sin\frac{n\pi x}{c}\right\}$$

$\theta = \gamma - u$

$\cancel{x = c - \xi}$

$\xi = c - x$

$\frac{x}{c} = 1 - \frac{\xi}{c}$

$$\theta - \gamma = \gamma\left\{1 - \frac{\xi}{c} + \frac{2}{\pi}\sum\frac{(-1)^n}{n}\, e^{-a^2\left(\frac{n\pi}{c}\right)^2 t}\,\underbrace{\sin n\pi\left(1-\frac{\xi}{c}\right)}_{\sin n\pi\frac{\xi}{c}}\right\}$$

$$\cancel{\theta = \gamma\left\{1 - \frac{\xi}{c} + \frac{2}{\pi}\sum\frac{(-1)^n}{n}\, e^{-a^2\left(\frac{n\pi}{c}\right)^2 t}\,\sin n\pi\frac{\xi}{c}\right\}}$$

$$\theta = \gamma\left\{2 - \frac{\xi}{c} + \frac{2}{\pi}\sum\frac{(-1)^{2n+1}}{n}\, e^{-a^2\left(\frac{n\pi}{c}\right)^2 t}\,\sin\frac{n\pi\xi}{c}\right\}$$

$$= \gamma\left\{2 - \frac{\xi}{c} - \frac{2}{\pi}\sum\frac{1}{n}\, e^{-a^2\left(\frac{n\pi}{c}\right)^2 t}\,\sin\frac{n\pi\xi}{c}\right\}$$

$$\cancel{\sum\frac{\sin nx}{n} = \frac{\pi - x}{2} \quad 1 - \frac{\xi}{c} - \frac{2}{\pi}\,\frac{\pi - \pi\xi}{2}}$$

$$\frac{d\varrho}{dt} = \frac{c_B\, s\, v}{e^{\frac{vh}{k}} - 1} \qquad v = \alpha \frac{d\varrho}{dt}$$

$$c_B = \frac{m-\varrho}{W}$$

$$\frac{d\varrho}{dt}\left[\frac{vh}{k} + \frac{1}{2}\left(\frac{vh}{k}\right)^2\right] = c_B\, s\, v$$

$$\frac{d\varrho}{dt}\left[1 + \frac{1}{2}\frac{\alpha h}{k}\frac{d\varrho}{dt}\right] = \frac{m-\varrho}{W}\,\frac{s\,k}{h}$$

$$\left(\frac{d\varrho}{dt}\right)^2 + \frac{2k}{\alpha h}\frac{d\varrho}{dt} = \cancel{\frac{2k}{\alpha}\,\frac{m-\varrho}{W}\,\frac{\alpha k}{h}} = \frac{m-\varrho}{W}\,\frac{2s}{\alpha}\left(\frac{k}{h}\right)^2$$

$$\frac{d\varrho}{dt} = -\frac{k}{\alpha h} \pm \sqrt{\frac{m-\varrho}{W}\,\frac{2s}{\alpha}\left(\frac{k}{h}\right)^2 + \left(\frac{k}{\alpha h}\right)^2}$$

$$= -\frac{k}{\alpha h}\left[-1 + \left(1 + \frac{m-\varrho}{W}\,2s\alpha\right)^{\frac{1}{2}}\right]$$

$$\frac{d\varrho}{dt} = -\frac{k}{\cancel{\alpha} h}\left[\frac{m-\varrho}{W}\,2s\cancel{\alpha} - \frac{1}{8}\left(\frac{m-\varrho}{W}\,2s\cancel{\alpha}\right)^2\right]$$

$$\theta = \gamma\left\{1 - \frac{\xi}{c} - \frac{2}{\pi}\sum \frac{1}{n}\, e^{-a^2\left(\frac{n\pi}{c}\right)^2 t} \sin\frac{n\pi\xi}{c}\right\}$$

$t = 0 \qquad \theta = 0$

$\xi = 0 \qquad \theta = \gamma$

$\xi = c \qquad \theta = 0$

$$\theta = \gamma\left\{\frac{\xi}{c} + \frac{2}{\pi}\sum \frac{1}{n}\, e^{-a^2\left(\frac{n\pi}{c}\right)^2 t} \sin\frac{n\pi\xi}{c}\right\}$$

$$\sum \frac{\sin n\alpha}{n} = \frac{\pi - \alpha}{2}$$

$$\frac{\xi}{c} + \frac{2}{\pi}\,\frac{\pi - \frac{\pi\xi}{c}}{2} = 1$$

$t = 0 \qquad \theta = \gamma$

$\xi = 0 \qquad \theta = 0$

$\xi = c \qquad \theta = \gamma$

$$\left.\frac{\partial\theta}{\partial\xi}\right|_0 = \frac{\gamma}{c}\left\{1 + 2\sum_{n=1}^{\infty} e^{-a^2\left(\frac{n\pi}{c}\right)^2 t}\right\}$$

[illegible] $\frac{2}{c}\, e^{-\frac{a^2\pi^2}{c^2}t}$ spadnie do wartości ε

$$e^{-\frac{a^2\pi^2}{c^2}t} = \frac{c}{2}\varepsilon$$

$$\frac{\partial c}{\partial t} = a^2\frac{\partial^2 c}{\partial x^2}$$

$$-\frac{\pi^2 a^2}{c^2}t = \lg\frac{c\varepsilon}{2}$$

$a^2 = k = 0.89$

$c = 3.65$

$$t = \frac{c^2}{\pi^2 a^2}\lg\frac{2}{c\varepsilon}$$

$$= \frac{c^2}{\pi^2 k}\left[\lg\frac{2}{c} - \lg\varepsilon\right]$$

Wł. Umiński

Pogadanki naukowe, Pioruny i Błyskawice. Warszawa Arct 108 str. 1897.

Zbiór różnych opowiadań o piorunach, po części faktów niezwykłych, po części wprost śmiesznie ~~naiwne~~ dziecinnych bajek ~~[illegible]~~ i podanie zupełnie bezkrytycznie i bez zrozumienia naukowego, za to wszystko silnie zabarwione sensacyjnością i ~~z [illegible]~~ ilustrowane okropnie marnemi ilustracyami. ~~[illegible]~~ (Takie książki ~~[illegible]~~ więcej szkody przynoszą umysłom niedojrzałym ~~niż [illegible]~~ pożytku.

—

...

M. Jamrógiewicz. Maciej Burza, Lwów nakł. Tow. Naucz. Szk. Wyż. 1906 str 34

Książka do czytania dla chłopców w wieku szkolnym, napisana pół żartem (lepiej niż popularnie) i zachęcająca swą treścią do zajęć fizycznemi w wieku szkolnym; ~~[illegible]~~ Sądzimy że i te niektóre urządzenia (możnaby zauważyć że niektóre z opisanych ~~robót~~ (np. (wiwar z rur blaszanych) są za trudne do wykonania w praktyce i może lepiej byłoby ograniczyć się na nieco skromniejszych, ale łatwiejszych do wykonania wynalazkach. Ale bardzo wysoko cenimy intencyę autora, które poważnie mierzy na uniknięcie wszystko. ~~[illegible]~~ (tą książkę [illegible] polecamy młodzieży szkolnej) ~~popularnie~~

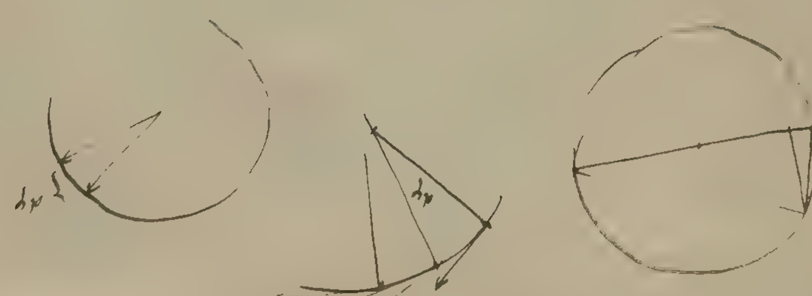
$$\int_0^\infty f'(y)\,[f(y)]^{n-1}\,dy = \frac{[f(y)]^n}{n}\Big|_0^\infty$$

$$\int_0^\infty e^{-\alpha y^2}\,dy\left[\int_0^y e^{-\alpha x^2}\,dx\right]^{n-1} = \frac{1}{n}\left[\int_0^\infty e^{-\alpha x^2}\,dx\right]^n = \frac{1}{n}\left[\frac{1}{2}\sqrt{\frac{\pi}{\alpha}}\right]^n$$

$$\frac{\partial}{\partial\alpha} \qquad -\int_0^\infty y^2 e^{-\alpha y^2}\,dy\cdot\left[\int_0^y e^{-\alpha x^2}\,dx\right]^{n-1} - \int_0^\infty e^{-\alpha y^2}\,dy\cdot(n-1)\left[\int_0^y e^{-\alpha x^2}\,dx\right]^{n-2}\cdot\int_0^y x^2 e^{-\alpha x^2}\,dx$$

$$\int_0^y x^2 e^{-\alpha x^2}\,dx = -\frac{y\cdot e^{-\alpha y^2}}{2\alpha} + \frac{1}{2\alpha}\int_0^y e^{-\alpha x^2}\,dx$$

$$\int e^{-\alpha y^2}\,dy$$



C_2H_4

$$\begin{array}{r} 64 \\ 4 \\ \hline 68 \end{array} : 29 = 2{\cdot}34$$

100
130

0·0013, 2·34

0·00·30

$$\frac{0{\cdot}2}{0003} = 66$$

1901

~~[illegible]~~

Poradnik dla samouków I (wyd. 2) Michalski & Heflich

Biernacki W. Fizyka i mechanika 48-77

Natanson W. Mechanika teoret. 78-82

Z phys. Ch. 39 p. 355-363 (1901) Ph. M. 2 p. 469-477 (1901)

Natanson O podw. załam. światła w ciecz. odkształc. Bull. Intern. (1901) 161-171

~~N...~~ O prawach zjaw. dyfuzji Rozpr. XLI p. (447-461) Bull. Int. (335-348) Rozpr. XLI (306-316)

~~N...~~ O prawach teorii ... Bull. (95-111)

Z ph. Ch. 38 p. 690-704 (1901); Ph. M. 2 p. 342-356 (1901) Rozpr. XLI (223-240)

~~N...~~ Wiadomości z Nauki Fizyki dla Szk. Nauk.

~~N...~~ Poglad na rodzaje zjawisk w nat. wszechświecie Przegląd P.

Lesicki Obliczanie statyczne murów oddzielonych Przegl. techniczny W. XXXIX 1-5

~~[illegible]~~ Kramsztyk Prawo i hypoteza w badaniu przyrody Bibl. Warsz. CCXLI p. 508

~~Pilkotycki Fizyka~~

XXVI p. 464 ~~Żórawski~~ XXVI p. 463 Szopiński Amat. Oświetlenie elektryczne

O pewnych zmiennych XXXVIII

O zastosowaniu metod wirowych XXXIX

Gorczyński O stosowalności wzorów dyspers. Wiad. mat. V (1-9)

Gosiewski Zarys teorji materiał. monadologji Przegl. filoz. IV (1-25)

XXVI p. 48 Silberstein Wstęp do dziedz. zjaw. elektrycznych I

Witkowski Uwagi o kilku ogólnych zasadach nauki fizyki Kosmos XXVI (1-14)

Zakrzewski O sile elektromot. Rozp. $\boxed{XXXIX}$ p. 258

Witkowski O ciekłem powietrzu Warszawa Drzewo i przebranie

Frank Z teoryi elektrodynamiki Spr. gimn. Fr. J. (1901)

Heinrich O metodologii nauk Przegl. fil. IV p. 257

Koelichen O zjaw. peryod. w elektrolizie Chem. Polsk. I p. 585

XXVI p. 465 Tomaszewski Promienie Röntg. Sprawozd. gimn. Sambor

Zausmer Termoregulator naftowy i uproszczona centryfuga Medyc. Warsz. XXIX (592)

Bigeleisen Rozwój pojęcia ruchu w mechanice Przegl. fil. Warsz. IV (306) V (17)

Kucharzewski O zadaniach filozofii techniki Przegl. techn. Warsz. XXXIX

~~Zakrzewski Lewi-Civita Metody rach. różn. i zast.~~ Prace m. fiz. XII. 1901

Ernst O nowym wzorze interpol. dla widma pryzmat. Prace XII p. 220

Gorczyński W sprawie jednostek elektr. i magn. Wiad. mat. V p. 274

Heinrich Krytyczny przegląd dotychczas. badań nad wrażeniami barw. Przyr. XL p. 1

" O stałości wrażenia zaledwie spostrzegalnego przy użyciu tonów czystych XL p. 41

XXVI p. 393 Olszewski Oznaczenie temp. inwersyi zjaw. Joule'a i Kelvina Rozpr. XLI p. 473

Silberstein Elektrolit. przenoszenie ... Kosm. XXVI p. 436

Smoluchowski O niektórych postępach na polu teoryi kinet. materyi Prace XII p. 112

Urbański O postępach w astron. i fizyce od najdawn. czasów aż do końca XIX Przewodnik nauk. i literat. XXIX p. 526, 648, 787, 883

Zawidzki Notatka historyczna o zjawiskach krytycz. Wiad. mat. Warsz. V p. 224

1902

XXVII Natanson — O rozchodzeniu się ruchów w płynach lepkich Rozpr. XLII p. 28 Bull. p. 19 (1902)

~~[illegible]~~ [illegible] Boltzmann Wiad. mat. VI p. 1

Denizot — Przyczynki do uzasadnienia matemat. drugiej zasady termodyn. Wiad. mat. VI 1902 (p. 56)

XXVII p. 342 Godlewski — O ciśnieniu osmot. niektór. roztw. — Bull p. 146 Rozpr. XLII p. 55

Gorczyński — Promieniowanie słon. i atm. ziemska Wszechświat XXI p. 161, 178

Natanson — O przewodn. ciepl. rozrz. gaz. Bull. 1902 p. 187 Rozpr. XLII (p. 70)

Rechan Fr. W. Spółczesne wyniki badań nad elektrolityczn. przewodn. [illegible] Przegląd tygodn. (163 stron)

[illegible]

p. 40 ~~Natanson~~ ~~Zmarszczki kołowe~~ ~~R XLII p. 392~~

XXVII p. 243 Zakrzewski K. O oscylacyi krążka w płynie lepkim Bull. (1902) p. 235

~~Kłapowski~~ Życie i prace księdza J. Rogalińskiego Rocz. T. N. XXVIII p. 113

Hortyński — Jonizacya gazów i rozkład atomu Sprawozd. gim. Chyrowsk. Przemyśl 1902

Janiów — Dyfuzya gazów i par Spr. gimn. Jarosł. 1902 p. 1.

Kraft — Badania doświadczal. nad skalą barw interf. Bull. 1902 p. 310 Rozpr. XLII p. 272

~~Natanson~~ Noiszewski — Znaczenie połyskiwania i załamania dla objawu Purkinjego Postęp okul. Kraków IV (1901) p. 49, 86, 121, 239

~~Rotarski~~ — Kryształy ciekłe Chem. Pol. II p. 467

~~Witkowski~~ — O prędkości głosu w pow. sprężonym Rozpr. XXXIX p. 1

Pirko Piotr rostw. Wiad. mat. VI p. 147

Heinrich Teorye i wyniki badań przyrody — I. Badań rzeczy? wzg? — Warnera (307 str.)

Stark 9 O cieknieniu, o rzymach, odrzutem popielowym? nadtrójt? Br. So. Warner? Aret

Walden & Centnerszwer Ciekły SO_2 jako rozpuszczalnik Wiad. mat. VI p. 213

XXVII p. 303 Wittkowski Sprostowanie? rad. elektr. ... v Żakop. Rozp. XLII p.

~~XXVII p. 3?? Dziewulski? Śnieżki jako podst. ...~~

~~Kostanecki Barwniki? pol...~~

~~XXVII p. ?? A. Dawid? Über ... Pendelpatten? v. Inles~~

~~XXVII p. ?? Wróblewski O teorii ...~~

XXVI p. 560 Laska Bericht ü. d. Erdbebenbeob. v. Lemberg Mitth. d. Erdbeben Com. d. Akad. Wien 1901 p. 64

~~XXVI p. 505 Silberstein Symbolische Integr. Annalen 6 p. 373 (1902)~~

XXVI p. 48 Zaniewski Über die Dampfdrucke binärer Flüssigkeitsgemische Pr. m. f. XIII p. 11

Denizot O pewnem zagadnieniu Eulera o wahadle Prace m. f. XIII p. 1 (1902)

Natanson O funkcyi dys. ... lepkich Rozp. XLII p. 399 (1902), Bull. 1902 p. 480

Natanson O odkształceniu ciężkich plastyczno-lepkich XLII p. 405, Bull. 1902 p. 494

~~Żakrzewski O ... ciężkie w płynie lept. Rozp. XLII p. 392~~

Kozłowski Ewolucya jako ogólna zasada stawania się Przegl. fil. V p. 368 (1902)

Olszewski Przyrządy do skropl. powietrza i wodoru Rozp. XLII p. 457, Bull. 1902 p. 619

Silberstein Teorya opadów fizycznych (Pierwszy zasad yawisk w rozni) Prz. fil. V p. 424

Zawidzki Studya doświ. nad przeźroczystością z składem pary podw. miesz. cieczy
Pr. mat. fiz. XIII p. 11

Curie Phys. Z. 2 p. 563.

Smoluchowski Ph. Z. 2 p 3, 65, 105; 3 p. 82, 129, 366

Gorczyński Über die Charakteristik d. Dispersionsformeln Ph Z 2 p. 205

Mitkiewicz W. Ph Z 2 p 747

~~Smol. Ph Z. 2 p. 307~~

~~Zakrzewski Ph Z. 2 p. 146~~

Żulkowski K. Ph Z. 3 p. 349

Zakrzewski Z f ph Ch. 14 p. 348–349 (1901)

(1904)

Żabicki Ü. d. Th. d. chem. Elemente Sprawozd. 7
odb. 61 str. 1903.

Witkowski A. Zasady Fizyki I (II [illegible]
II₂ [illegible]

Kowalski [illegible] 1904.
~~[illegible] Teorya chemii~~ ~~[illegible]~~

Estreicher T.

Dickstein Ewolucya mat. Wszech.

Witkowski Tablice mat. fiz.

Dorsett Zasady Mechaniki 1903
Krótki zys fizyki 1903
— Warburg 1903
Kramsztyk Wiad. pom. z fizyki 1903
~~Szymański [illegible] Fizyka 1902~~
Wiad. mat. p. 115

Deniset Przyczynek do Th. chem. galw. odw. Wiad. 1904 p. 47

Kucharzewski [illegible] Wiad. 1904 p. 03

Danielewicz [illegible] p. 29

77

~~[illegible]~~

Natanson Teorya [illegible] Przegl. fil.

Danielewicz Metoda [illegible] Wiad. 1904 do str. 186, XII
(Kramsztyka)

~~[illegible] 135~~

Huber M.T. Tomiczewski: Wszechświat 1903 Wiad mat p 135

Niemiecko polski słowniczek matematyczny Turyn 1904 p. 37 i 19-

Z filozofii nauk przyrodniczych Wł. Rudzki

1904 Wszechświat

Antoniewicz 1904 O promieniotwórczej materyi Przyroda 1904

Loria Poglądy i teorye fiz. współ. — Pollonsko 1904

Stark-Dunner Radium i zmowy atom. chemicz.

Mieczski Die Gesamtheit ... Not Df 1904 p 11-18

~~Huber Zur Theorie ... Ann Ph. 14 p 153-163 (1904)~~

Zawidzki O „Regnault'sche Gesetz" ... Ztsch phys. Ch. 46 p 21-29

" Ü. amphoteren ... Kakodylsäure

Ber Chem Ges. 37 p 153-154

" ... $NaNO_3 + AgNO_3$ Z. phys. Chem 47 p 721-728 (1904)

Zakrzewski Urbański Nikol Wiad p 145

Ernst Obserwacye gwiazd zmiennych Prace mat 1904 p 29-46

Litwiński O podst. teoryi wytrzymałości Prace mat. 1904 p 47-68

Łomnicki O metodzie podobieństwa dynam ... " p. 115-133

Ernst O wyznaczeniu paralaksy promieniowej ... p 135-160

Mydlarski Prawo B. M. : Kosmos 29 p 302

Zaremba Journal d. math. Liouville 10 p 395-444 (1904. Sur l'intégrale ... [illegible]

et Edmond ... [illegible] ... [illegible]

~~Huber O najważniejszych technicznie wynikach teoryi hydrokinetyki~~

~~Czasopismo techn. Lwów~~ XXI (1903) p. 47-49, 61-62, 73-74, 84-85, 99-100, 117-118, 134-135

Natanson ~~O stopniu przybliż. pew... l.~~ Bull. 1903 (283-311) Rozp. 43 (195-222) 1903

~~O zastosow. rów. Lag.~~ " (268-283) | ..(179-194)..

~~O teoryach materyi wykład...~~ Przegląd filoz. 147, 465-478 (1903)

~~Uwagi nad teoryą zjawiska złudzenia~~ Rozp. 43 (595-615) 1903

Bull. Int. (767-787) 1903

Orłowski Teorya naukowa latawca Wszechświat 22, 257-261, 276-279, 278-281

~~Smoluchowski Zjawisk akustycznych~~ Rozp. 43 (1903). 71-109

Stodółkiewicz O zmianach natężenia siły ciężkości Przegl. techn. Warsz. 41 p. 392-393 (1903)

Witkowski Nowe długości metryczne i ich stos. do miar angielskich i dawnych polskich Prz. t. W. 41 p. 81-85

Zaremba ~~O pewnem uogólnieniu kl. t.~~ Bull. 380-403 Rozp. 43, 223-246

~~O podw. zbior.~~ Bull. 403-422, Rozp. 43, 247-266

~~O pewnej postaci dost.~~ Bull. 594-614 Rozp. 43, 482-502

~~Zasada minimów wyłącz.~~ Bull. 614-621, Rozp. 43, 503-510

~~O pewnem zagadn. hydrodyn.~~ ~~Rozp. 43 247~~

~~Olszewski Nowy przyrząd do skropl. wodoru~~ Bull. 1903, 241-246 Rozp. 43, 142-147

Wied. Ann.

Duhem Ewolucya mechaniki Wiad. mat. 7, 244-288

~~Huber Z teoryi zginania belki~~ " 289-292

Estreicher Punkty topl. tlenu i azotu Bull. 831-844

~~Kowalski, Zdanowska Nowa metoda mierz. oporu płyn. Rozp 43, 573-594~~

Bull. 783-794

Tomaszewski-Kowecki Fizyka Z.

Bauer-Toll Szybkość rozpuszcz. Rozp 440-481, Bull. 555-594

Stodółkiewicz Kilka uwag o [illegible] Przegl. tech. Warsz. 41 (510-512)

Rotarski Kryształy ciekłe Chem. pols. 3, 783-786

~~Smoluch. Endosmoza elektr. Rozp. 43, 110-129~~

~~Natanson & [illegible]~~ Ph. Z. 4, 541-543 (1903)

~~Witkowski Elektr. przegl. t. 147, 312-323 (1903)~~

Heinrich O funkcyi [illegible] R. 289-308